LOGIC

HIGHER SECONDARY
FIRST YEAR

Vol - II

TAMILNADU TEXTBOOK SOCIETY

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Vol. II.

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TAMILNADU TEXTBOOK SOCIETY
MADRAS

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First Edition — 1978

(Prepared by the Committee constituted under the authority of the Government of Tamilnadu)

Price : Rs. 2-00

This book has been printed on 60 G.S.M. paper.

Printed at :
Neo Art Press, Madras-600 002

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Chapter V

THE HYPOTHETICAL SYLLOGISM

Section 1. The Hypothetical proposition

A hypothetical proposition is a conditional statement in which the condition is stated in the "if S, then P", form. The condition is in the form of 'cause and effect' relation.

Example :

If a man takes poison, he will die.

↓	↓
states the cause or condition	states the effect asserted under the condition

The clause which states the cause is called the *antecedent* and the clause which states the effect is called the *consequent*.

Section 2. The Hypothetical syllogism

(a) *Structure*

A hypothetical syllogism has a hypothetical proposition for its major premise, a categorical proposition for its minor premise and a categorical proposition of its conclusion. Hence, it is sometimes called *mixed hypothetical syllogism*.

(b) *Rule*

The rule of the hypothetical syllogism is : *Either affirm* the antecedent in the minor premise *Or deny* the consequent in the minor premise.

It is usually said that the rule of the hypothetical syllogism is in the form of a disjunctive proposition. These are not two different rules applicable to one and the same hypothetical syllogism but only one rule stated in a disjunctive form. That is, affirm the antecedent in the minor premise *or* deny the consequent in the minor premise.

By 'affirming' we mean 'stating' what is given and by 'denying' we mean 'contradicting' what is given.

(Note : If we want to deny the consequent we must do so by the contradictory and not by the contrary).

(c) *Proof of the rule* (or the rationale of the rule)

In a hypothetical proposition the antecedent states the cause and the consequent states the effect.

Example:

If a man takes poison, he will die. If the cause is present the effect must be present. Therefore, by affirming the antecedent we can come to a conclusion. But we cannot infer that if a man does not take poison, he will not die. For death has many causes like old age, disease, drowning, etc. similarly we cannot argue that if a man dead his death is due to taking poison. For, poisoning is not the only cause of death. Thus from the affirmation of the consequent we cannot infer the affirmation of the given antecedent. But we can argue that if a man is not dead he has not taken poison. Hence the rule: either affirmthe antecedent in the minor premise or deny the consequent in the minor premise.

The rationale of the rule may be represented as follows (√ means affirmation ; × means denial; and ? means no conclusion can be drawn.)

If a man takes poison — *he will die.*

↓ — ↓

antecedent — consequent

√ ⟶ √

× ⟶ ?

× ⟵ ×

? ⟵ √

The entire theory of the hypothetical syllogism may be summed up as follows.

To affirm the antecedent is to affirm the consequent, to deny the consequent is to deny the antecedent; but to deny the antecedent is not to deny the consequent; and to affirm the consequent is not to affirm the antecedent.

Exception to the rule

But there are cases in which the given antecedent is the ***only antecedent or the sine qua non*** of the consequent. In such cases, we can proceed to a conclusion from the affiirmation or denial of both the antecedent and the consequent.

Example :

If it is a six legged creature,		*it is an insect*
↓		↓
Antecedent		Consequent
✓	←——————→	✓
×	←——————→	×

(d) *Valid moods or kinds*

Hypothetical syllogisms are of two forms or kinds either as we affirm the antecedent or deny the consquent in the minor premise.

(i) *Modus Ponens* or Constructive Hypothetical syllogism*

In this mood the antecedent is affirmed in the minor premise thereby the consequent is affirmed in the conclusion.

If a man takes poison, he will die.

This man has taken poison.

∴ This man will die.

(ii) *Modus Tollens** or Destructive Hypothetical syllogism*

In this mood the consequent is denied in the minor premise and thereby the antecedent is denied in the conclusion.

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*'Ponere' in Latin means 'laying down a truth'.
**'Tollere' in Latin means 'removing an error'.

If a man takes poison, he will die.

This man is not dead.

∴ **This man has not taken poison.**

(e) *Fallacies*

According to the rule of the hypothetical syllogism we must *either* affirm the antecedent in the minor premise *or* deny the consequent in the minor premise.

Violation of this rule will lead to either of the following fallacies :

(i) *The fallacy of denying the antecedent*

If a man takes poison he will die.

This man has not taken poison.

∴ This man will not die.

(ii) *The fallacy of affirming the consequent*

If a man takes poison, he will die.

This man is dead.

∴ This man must have taken poison.

Note : But if the antecedent is the only antecedent of the consequent all the four forms represented below are valid :

1. If this is water, it is H_2O.

This is water.

∴ This is H_20.

2. If this is water it is H_2O.

This is not water.

∴ This it not H_2O.

3. If this is water, it is H_2O.

This is H_2O.

∴ This is water.

4. If this is water, it is H_2O.

This is not H_2O.

∴ This is not water.

Exercises

Identify and examine the following arguments :

Note :

A. First restate the argument in the proper form:

(a) Pick out the conclusion; leaving space for the premises write the conclusion first. (It need not be put in the logical form).

(b) The hypothetical proposition in the argument be the major premise. Put it as it is.

(c) The remaining proposition will be the minor premise.

B. Examine the argument :

(a) Look at the minor premise and note whether it refers to the antecedent or the consequent.

(b) If the minor premise refers to the antecedent and if it affirms the antecedent the argument is valid. (state the mood as 'Modus Ponens') If it denies the antecedent the argument is invalid committing the fallacy of denying the antecedent.

(c) If the minor premise refers to the consequent and if it denies the consequent the argument is valid (state the mood as Modus Tollens). If it affirms the consequent the argument is invalid commiting the fallacy of affirming the consequent.

(d) If the antecedent is the only antecedent or the sine qua non of the consequent even when the rule is violated the argument will be valid.

(e) State the mood only when the argument is valid; not otherwise.

1. If you eat spicy foods, you will get ulcers. You have got ulcers. Therefore, you must have taken spicy foods.

Restatement :

If you eat spicy foods, you will get ulcers.

You have got ulcers.

∴ You must have taken spicy foods.

This hypothetical syllogism commits the *fallacy of affirming the consequent*.. Here, the consequent is affirmed in the minor premise. According to the rule, the consequent must be denied in the minor premise. Since the argument violates the rule it is invalid.

2. If the world exists, God exists; God must exist, because the world exists.

Restatement :

If the world exists, God exists.

The world exists.

Therefore, God must exist.

This hypothetical syllogism is perfectly valid, the mood being Modus Ponens. Here the antecednet is affirmed in the minor premise. According to the rule, the antecedent must be affirmed in the minor premise. Since the argument does not violate the rule it is perfectly valid.

Note : The student is requested to construct as many valid and invalid syllogisms as possible using the following hypothetical propositions as the major premise of the argument. Using each we can have two valid forms and two invalid forms.

1. If you are amused, you will smile.
2. If you follow your conscience, you will be happy.
3. If one does not run, he loses the race.
4. If the plant is not watered, it will die.
5. If x is right, y is a liar.
6. If only men lay down their weapons, will they have peace. (This is a case of sine qua non).
7. If electricity is turned off, the lights will go out.
8. If a man is a politician, he is interested in securing votes.
9. If a man loves his neighbour, he will help him.
10. If the country wishes to avoid war, it will favour disarmament.
11. If I oversleep, I shall be late to the class.
12. If it rains, water need not be sprinkled.
13. If all men are born equal, slavery is unjust.
14. If a soup is cold, then it is tasteless.
15. If money grows on trees, there will be no economics.
16. If the road is wet, then it is slippery.
17. If that is the mail train, it is an express.
18. If a book has artistic merit, it will be widely read.
19. If there is progress, there is change.
20. If the sun is hot, the grass is dry.
21. If the patient has fever, he is infected.
22. He will have a breakdown, if he is not careful.
23. Wisemen suffer, whenever politicians rule (Whenever = if).

24. If a window is not open, it is shut.

25. If you do not study, you will fail.

26. If today is Tuesday, tomorrow is Wednesday.

27. If it barks, then it is a dog.

28. If he is not a thief, you will get your purse back.

29. If that bill is passed, rents will raise.

30. If a man is sick, he sends for the doctor.

31. If a man has a free will, he is responsible for his actions.

32. If a metal is heated, it expands.

33. Fallacies would be only excusable, if they are unavoidable.

34. If wages rise, prices increase.

35. If one is guilty, he trembles with fear.

36. If he were clever, he would see his mistake.

37. If a man is married, he is unhappy.

38. Only if you break the law, you are imprisoned.

39. If there is a total eclipse of the sun, the streets are dark.

40. If the country goes to war, the unemployment problem can be solved.

41. If trains are late, the railway stations are crowded.

42. If a lecturer reads from notes, he is a bore.

43. If a book is banned, everybody wants to read it.

44. If you eat too much ice cream, you will vomit.

45. If you take quinine, you will be cured of malaria.

46. If a man talks of rights, he must have discharged his duties.

47. If one is truly musical, he will be applauded.

48. If logic is useless, it can be ignored.

49. If I can perceive God, I can believe in Him.

50. If a student take notes, his mind is distracted.

51. If a man is hungry, he is irritable.

Questions

1. Explain and illustrate the hypothetical syllogism. What are its two valid moods? Name and illustrate them.

2. What are the fallacies which may arise in a hypothetical syllogism? Explain, with examples, why they are fallacies.

3. State and explain the rule of the hypothetical syllogism.

Chapter VI

THE DISJUNCTIVE SYLLOGISM

Section 1. The disjunctive proposition

A disjunctive proposition is a conditional statement. In it the condition is in the form of alternative predications—S is either P or not P. A disjunctive proposition may contain two or more alternatives. A true or perfect disjunction signifies 'either......... or........ and not both'.

A disjunctive proposition should satisfy two conditions in order to be valid or perfect. They are :

a. the alternatives must be mutually exclusive.

b. the alternatives must be collectively exhaustive.

(a) The alternatives must be mutually exclusive

This means that the alternatives stated should exclude one another. In other words *one alternative should shut out the other alternative.* They must be related as contradictories. Between alternatives there should not be any middle ground. The alternatives must have such a nature that they cannot be predicated of a thing at the same time. If one alternative is true, the other alternative must be false, and if one alternative is false the other alternative must be true.

Example :

'Categorical propositions are either universal in quantity or particular quantity.'

It cannot be both. That is certain. If a categorical proposition is universal, it cannot be particular. If it is particular, it cannot be universal. If it is not particular it must be universal. Thus universal and particular exclude each other.

Let us take an example in which the alternatives are not mutually exclusive.

Example:

> Categorical propositions are either universal or affirmative.

Here the alternatives do not exclude each other. For a categorical proposition can both be universal and affirmative.

(b) The alternatives must be collectively exhaustive

A disjunctive proposition should state all the possible alternatives about a subject. The alternatives stated must exhaust or complete all the possibilities. That is *no alternative should be omitted.* If any alternative is omit ed the disjunctive proposition is imperfect.

Example:

> Categorical propositions are either universal in quantity or particular in quantity.

Here we have stated all the alternatives which a categorical proposition (on the basis of quantity only) can be. That is, inrespect of quantity, there are only two possible alternatives which a categorical proposition can be. These two universal and particular, complete or exhaust together all the alternatives. Therefore the alternatives are collectively exhaustive.

Let us take an example in which the alternatives are not collectively exhaustive.

Example:

> A book deals with either logic or physics.

Here the alternatives are not exhaustive or complete. For the book may deal with chemistry, or mathematics, or botany or psychology etc. These other alternatives are not stated here. Logic and physics do not together exhaust all the possibilities about a book. Hence this is a case of imperfect disjunction.

Hence the alternatives which we state about the subject of a proposition must first exclude one another and then must together make up all the possible alternatives.

Other examples of disjunctive propositions where the alternatives are mutually exclusive and collectively exhaustive :

1. The signal light is either red or green.
2. Men are either bachelors or married.
3. A given number is either odd or even.
4. According to relation, propositions are either categorical or conditional.
5. A christian is either a protestant or a catholic.
6. A man who is alive is either inside the house or outside the house.

Section 2. The disjunctive syllogism

(a) Structure

A disjunctive syllogism consists of a disjunctive proposition for its major premise, a categorical proposition for its minor premise and a categorical proposition for its conclusion.

(b) Moods or types or kinds

The major premise is a disjunctive proposition consisting of alternatives. The minor premise affirms or denies any alternative of the disjunctive major. If the minor premise affirms one alternative the conclusion denies the other alternative. If the minor premise denies one alternative the conclusion affirms the other alternative.

Thus there are two types of disjunctive syllogisms :

(i) *Modus Ponendo Tollens* is a disjunctive syllogism in which the minor premise affirms one alternative and the conclusion denies the other.

A christian is either a protestant or a catholic.
John who is a christian is a protestent.
∴ John is not a catholic.

(ii) *Modus Tollendo Ponens* is a disjunctive syllogism in which the minor premise denies one alternative and the conclusion affirms the other.

A christian is either a protestant or a catholic.
John, who is a christian, is not a catholic.
∴ John is a protestant.

(c) Rules

The validity of a disjunctive syllogism depends on the validity of the disjunctive major premise. Only if the major premise is in proper disjunction the argument deserves the name disjunctive syllogism. The disjunctive major premise will be a case of perfect or proper disjunction only when it satisfies the following conditions.

(i) The alternatives must be mutually exclusive.
(ii) The alternatives must be collectively exhaustive.

(d) Fallacies

A fallacy of disjunctive syllogism is due to imperfect or improper disjunction. If the alternatives in the major premise, are not mutually exclusive, the argument commits the fallacy of *imperfect disjunction due to non-exclusive members.* If the alternatives in the major premise are not collectively exhaustive, the arument commits *the fallacy of imperfect disjunction due to non-exhaustive members.*

(i) Imperfect disjunction due to non-exclusive members.

An Indian is either a Tamilian or a politician
Shri Maniam is a politician
∴ He is not a Tamilian.

(ii) Imperfect disjunction due to non-exhaustive members.

Trees are either mango trees or coconut trees.
The tree in front of my house is not a mango tree.
∴ It is a coconut tree.

Exercises

Identify and examine the following arguments :

A. ***Note :*** *First restate the argument in the proper form.*

(a) Pick out the conclusion first. (It need not be put in the logical form).

(b) The disjunctive proposition in the argument will be the major premise. Put it as it is.

(c) The remaining proposition will be the minor premise.

B. Examine the argument.

(a) Irrespective of the fact that the argument be valid, or invalid state the mood of syllogism.

(b) Look at the major premise. If the alternatives are mutually exclusive and collectively exhaustive, the argument is valid.

(c) If the alternatives are not mutally exclusive the argument commits the fallacy of Imperfect Disjunction due to non-exclusive members; if the alternatives are not collectively exhaustive the argument commits the fallacy of Imperfect Disjunction due to non-exhaustive members.

1. **Students are either intelligent or industrious. Rama who is industrious must not be intelligent.**

Restatement :

Students are either intelligent or industrious.
Rama is industrious.

∴ Rama is not intelligent.

This disjunctive syllogism is of the mood Modus Ponendo Tollens. Here, the alternatives in the disjunctive major premise are not mutually exclusive. According to the rule the alternatives must be mutually exclusive and collectively exhaustive. Since

the argument violates the rule it is a case of imperfect disjunction due to non-exclusive members.

2. This proposition must be conditional, for propositions, according to relation, are either categorical or conditional.

Restatement :

According to relation, propositions are either categorical or conditional.
This proposition is not categorical.
∴ This proposition is conditional.

This disjunctive syllogism is of the Mood Modus Tollendo Ponens. Here the alternatives are both mutually exclusive and collectively exhaustive. According to the rule the alternatives must be mutually exclusive and collectively exhaustive. Since the argument does not violate the rule, it is a case of perfect disjunction.

(Note : The student is requested to construct a disjunctive syllogism either in the mood Modus Ponendo Tollens or in the mood Modus Tollendo Ponens using the following disjunctive propositions as the major premise and then examine the arguments).

1. The will is either free or not free.

2. Your sickness is caused by overwork or by too little sleep.

3. The ladder slipped either because I was too heavy for it or because it was made of too light a wood.

4. Either he does not know how to read or he does not know how to spell.

5. Animals are either male or female.

6. Either God is unjust or no man is eternally punished.

7. A door must be either open or shut.

8. We shall either have a leader or be powerless

9. Either Americans are not peace-loving or they will insist upon neutrality.

10. Either there is no mail for us this morning or the postman has not yet come.

11. Either a man is slave to money or he is a slave to change.

12. Man is the result either of creation or of evolution.

13. All energy is either kinetic or potential.

14. Either a man is married or he is a bachelor.

15. Either she loved me or she was deceiving me.

16. A coloured surface is either black or white.

17. A living being is either mortal or immortal.

18. Either eat your cake or have it.

19. An action is either good or bad.

20. Our team must either win or lose.

21. The world is either contingent or self-explanatory.

22. Either we avoid war or civilisation is lost.

23. Either he violated the law or he was arrested unjusty.

24. Either a book is for adults or for children.

25. A line is either straight or curved.

26. Either he is born in April or in May.

27. A man is either indifferent or forgetful.

28. A successful man has either ability or influence.

29. Organisms are either vertebrates or invertebrates.

30. Trees are either mango trees or tamarind trees.

31. Akbar was either a great statesman or a military genius.

32. Tagore was either a poet or a philosopher.

33. The walking stick is either ornamental or useful.

34. A professor is loved either for his scholarship or for his sportsmanship.

35. Logic is either a science or an art.

36. Flowers are liked either for their beauty or for their smell.

37. Either the strategy was bad or the execution was careless.

38. A has either been badly taught or has been himself lazy and indifferent.

39. Students are either athletic or bright.

40. Either we visit the dentist or have trouble with our teeth.

41. A baby is either a boy or a girl.

Questions

1. Explain and illustrate the disjunctive syllogism.

2. Distinguish between perfect and imperfect disjunctive propositions.
   Explain the moods of a disjunctive syllogism.

3. What are the fallacies which may arise in a disjunctive syllogism ? Explain, with examples, why they are fallacies.

Chapter VII

DILEMMA

Section 1. What is a dilemma?

We speak of a man being in a dilemma when there are only two courses open to him and both lead to unpleasant consequences. In logic it is used as a controversial weapon.

A dilemma consists of:

1. a compound hypothetical major premise;
2. a disjunctive minor premise; and
3. a disjunctive or categorical conclusion.

A dilemma is said to be *'simple'* if there is a common clause in the major premise.

A dilemma is said to be *'complex'* if there is no common clause in the major premise.

A dilemma is said to be *'constructive'* if the antecedents of the major premise are affirmed in the minor premise.

A dilemma is said to be *'destructive'* if the consequents of the major premise are denied in the minor premise.

Section 2. Kinds of dilemma

dilemmatic arguments are of four kinds.

I. *Simple constructive dilemma*

Symbolic example :

If A is B, C is D; If E is F, C is D
Either A is B or E is F.
Therefore C is D.

Concrete example :

If an elected member acts in accordance with his own judgement he will be criticised ; If he acts in accordance with the views of his constituents he will be criticised.

He acts either in accordance with his own judgement or in accordance with the views of his constituents.

Therefore he will be criticised.

II. *Simple destructive dilemma*

Symbolic example :

If A is B, C is D ; If A is B, E is F.
Either C is not D or E is not F.
Therefore A is not B.

Concrete example :

If a thing moves it moves where it is ; If a thing moves it moves where it is not.

Either it cannot move where it is, or it cannot move where it is not.

Therefore a thing cannot move.

III. *Complex constructive dilemma*

Symbolic example :

If A is B, C is D ; If E is F, G is H.
Either A is B or E is F.
∴ Either C is D or G is H.

Concrete Example :

If he wins the case, he must pay me by the terms of the original contract. If he loses the case, he must pay me by the judgement of the court.

Either he wins or loses the case.

Therefore, he must pay me either by the terms of the original contract or by the judgement of the court.

IV. *Complex destructive dilemma*

Symbolic example :

If A is B, C is D; If E is F, G is H.
Either C is not D or G is not H.
Either A is not B or E is not F.

Concrete example :

If an officer is obedient, he carries out the orders; If he is intelligent he understands them.
This officer either does not carry out the orders or does not understand them.

Therefore, this officer is either not obedient or not intelligent.

Conditions under which a dilemmatic argument is valid :

1. The consequents in the major premise must necessarily follow from their respective antecedents.

2. The alternatives in the disjunctive minor premise must be mutually exclusive and collectively exhaustive.

3. *Either* affirm the antecedents of the major premise, in the minor premise and then affirm the consequents in the conclusion.

4. *Or* deny the consequents of the major premise in the minor premise and then deny the antecedents in the conclusion.

Section 3. Methods of attacking dilemmatic arguments

I. We may "take the dilemma by horns."

This method of attacking a Dilemma consists in showing that the consequents in the major premise do not necessarily follow from their respective antecedents. In this method, we question the truth of the major premise.

Example :

If a man is single he would be unhappy, because he has no one to take care of him ; If he is married he would be unhappy because he has to take care of his wife.

A man is either single or married.

Therefore he would be unhappy either because there is no one to take care of him or he has to take care of his wife.

In the major premise there is no necessary connection between the antecedents and the consequents. For there are people who are single and yet quite happy. Married people are also happy. Therefore the dilemma is false.

II. We may "escape between the horns of a dilemma"

This is simply to point out that the alternatives presented in the minor premise are not exhaustive and that there are one or more other possibilities left unmentioned.

Example :

If students are intelligent, prizes are unnecessary ; If students are dull, prizes are useless.

Students are either intelligent or dull.

Therefore prizes are either unnecessary or useless.

This dilemma is false, because, the alternatives in the minor premise are not collectively exhaustive. A large majority of students are neither intelligent nor dull. Prizes may be effective in their cases.

III. Rebuttal

Rebuttal consists in constructing a counter dilemma whose conclusion is the very opposite of the original conclusion. Rebuttal is a method of paying back the opponent in his own coin. The counter dilemma may be itself invalid as the original dilemma. Yet it is useful to expose the weakness of the original dilemma. It is a striking rhetorical device for attacking the original dilemma.

Original dilemma

If students are idle, examinations are useless; If students are industrious examinations are unnecessary.
Either students are idle or industrious.
Therefore examinations are either useless or unnecessary.

Counter dilemma

If students are idle, examinations are not unnecessary ; If students are industrious examinations are not useless.
Students are either idle or industrious.
Therefore, examinations either not unnecessary or not useless.

Symbolic example :

If A is B, C is D ; If E is F, G is H.
Either A is B, or E is F.
Therefore, either C is D or G is H.

Symbolic example :

If A is B, G is not H ; If E is F, C. is not D.
Either A is B or E is F.
Therefore, G is not H or C is not D.

The formula of rebuttal is : Change the quality of the Consequents and transpose them.

Exercises

Identify and examine the following :

1. No honest lawyer will plead for an accused person. For the accused is either guilty or innocent, If he is guilty, he ought not to be defended and if he is innocent, it must be apparent to the judges.

2. There can be no ignorance. For if a man knows what he is ignorant of, then he is not ignorant of it. If he does not

know what he is ignorant of, then where does ignorance exist?

3. If a country is prosperous, people will be loyal; if a country is prosperous people will be happy. Either the people are not loyal or they are not happy. Therefore, the country is not prosperous.

4. If the population increases there will be food problem; if the population decreases there will be labour problem. Either there is no food problem or there is no labour problem. Therefore, either there is no increase in population or there is no decrease in population.

5. If the authorities are lenient, they are criticised; if they are strict, they are criticised, Either the authorities are lenient or they are strict. Therefore the authorities are bound to be criticised.

6. The Alexandrian Library should be burnt. For if the books in the library teach the same doctrine as the Koran, they are superfluous If they teach something different from Koran they are pernicious.

7. If a thing advertised is good, people would go in for it without their attention being drawn to its merits. If a thing advertised is not good it is immoral to praise its qualities. Hence advertisement is either not necessary or not moral.

8. If the student takes notes his mind is distracted; if he does not his memory may let slip some important points. The student either takes notes or does not.

9. It is useless to go canvassing. for either people intend to vote for your candidate in which case canvasssing is superfluous; or they do not intend to vote. in which case canvassing is ineffective.

10. If there is no control of the press, sedition grows rife, if there is control there is intolerable oppression; any way the press as an institution is an unmitigated evil.

11. It is useless to give advice, for if a man is wise, he will not need it; and if he is not wise, he will go his own way.

12. There is no harm in allowing boys to climb trees. If they are confident they are safe. If they are nervous, they will not climb high enough to run a risk.

13. If students are idle, examinations are unaviling; and if they are industrious examinations are superfluous. Accordingly as all students are either idle or industrious, examinations are useless.

14. If some who have studied Logic cannot argue well, it is a useless study; if some who have not studied Logic can reason well, it is an unnecessary study. In any case logic is of no use.

15. It is useless to take medicine, for if I am destined to diel no medicine can cure me. If I am distined to recover no medicine is needed.

16. Protective laws should be abolished for they are injurious if they produced scarcity and they are useless if they do not.

17. If a man stays in the room, he will be burnt to death; if he jumps out of the window he will break his neck; either the man stays in the room or he jumps our of the window. Therefore, either he will be burnt to death or he will break his neck.

18. If taxation is heavy, people will become poor as capital formation is discouraged; and if taxation is not heavy no constructive national schemes can be undertaken. Thus the predicament is inescapable that either the people will become increasingly poor or no constructive national schemes can be undertaken.

19. Motor cycling is an unsatisfactory way of seeing the country. If the roads are good, you travel too fast to enjoy the scenery. If they are bad, you are too uncomfortable to look at it.

20. If I love my wife, my mother-in-law likes me; if I hate her, my mother likes me; either I love my wife or I hate her. In either case I am liked.

Questions

1. State and explain the four kinds of dilemma.
2. What is a dilemma? Explain the different ways of attacking the dilemmatic arguments.
3. What is meant by rebuttal? Explain with examples.

Chapter VIII

FALLACIES

Sec. 1. What is a fallacy?

Sec. 2. Errors in interpretation.

Sec. 3. Mistakes in reasoning.

Section 1. What is a fallacy?

A fallacy is an argument which appears to be conclusive when it is not. A fallacy is a wrong or unsound inference. It is a violation of a logical principle disguised under a show of validity. The word 'fallacy' comes from the latin *fallacie* meaning deception. Plato says that arguments, like men, are often pretenders.

Aristotle divides fallacies, by dichotomy, into (a) those which arise out of ambiguity of language (fallacies *in dictione*) and (b) those which are not the result of such ambiguity (fallacies *extra dictionem*). He enumerates six fallacies *in dictione* and seven *extra dictionem* as follows:

A. *Fallacies in dictione:* (1) Equivocation (2) Amphiboly, (3) Composition, (4) Division, (5) Accent and (6) Figure of speech.

B. *Fallacies extra dictionem*: (1) Accident, (2) Secundum Quid, (3) Ignoratio Elenchi, (4) Petitio Principal, (5) Consequent, (6) Non causa Pro causa and (7) Many questions.

We may classify logical fallacies as follows:

Section 2. Errors in interpretation

This class of fallacies result from imperfect understanding of the meaning of propositions.

(i) Illogical obversion

(i) Illogical obversion results if the rules of obversion are not observed. Rules of obversion:

FALLACIES

ERRORS IN INTERPRETATION

1) Illogical obversion
2) Illogical conversion
3) Amphiboly
4) Fallacy of Accent

MISTAKES IN REASONING

FORMAL FALLACIES

Categorical Syllogism

1) Four Terms
2) Undistributed Middle
3) Illicit Major
4) Illicit Minor
5) Two Negative Premises
6) Two Particular Premises

Hypothetical Syllogism

7) Denying the Antecedent
8) Affirming the consequent[7]

Disjunctive syllogism

9) Imperfect Disjunction

MATERIAL FALLACIES

FALLACIES OF EQUIVOCATION

1) Ambiguous and shifting terms
2) Composition
3) Division
4) Accident
5) Dilemmatic Fallacy

FALLACIES OF PRESUMPTION

1) Petitio Principii
2) Complex Question
3) Irrelevant Conclusion
4) Non-sequitur

(a) Do not change the quantity of the original proposition (b) Do not change the subject of the original proposition (c) change the quality of the original proposition (d) take the contradictory of the original predicate.

Instances of illogical obversion :

(1) Some men are not happy.
∴ Some men are unhappy.

This inference is false because it asserts not merely the absence of happiness but the presence of positive misery. The logical contradictory of the predicate should be used.

(2) Some men are not rich.
∴ Some men are poor.

This inference is fallacious because 'rich' and 'poor' are contraries and there are many intermediate stages between them.

(3) Honesty is always a good policy.
∴ Dishonesty is always a bad policy.

This is called *material obversion.* This is not a formal process of inference but is based on experience. But in deduction we pay attention to the formal aspect of an inference. We are not interested in the truth or otherwise of the second proposition. What we should see is whether it is the implication or meaning of the first proposition. Here, in the second proposition, the subject and the predicate terms are the contraries and not the contradictories, of the original terms. Again we have violated the rules of obversion by changing the subject.

Examples of material obversion :

4. No strangers are allowed to vote.
∴ All citizens are allowed to vote.

5. Wealth is desirable.
∴ Poverty is undesirable.

6. War is productive of evil.
∴ Peace is beneficial.

7. Knowledge is good.
∴ Ignorance is bad.

8. Warmth is agreeable.
∴ Cold is disagreeble.

9. A false weight is an abomination to the Lord.
∴ A just weight is A is delight.

10. An upright minister gives public confidence.
∴ A shuffling minister causes distrust.

(ii) Illogical conversion

Illogical conversion arises generally if proposition A is converted simply instead of by limitation or if proposition O is converted.

Instances of illogical conversion of proposition A :—

1. All cats are animals.
∴ All animals are cats.

2. All pious people go regularly to church.
∴ Regular church going is a sure sign of piety.

Instances of (illogical) conversion of O proposition:—

1. Some men are not honest.
∴ Some honest beings are not men.

2. Some who sit for an examination do not pass.
∴ Some who pass an examination are not persons who sit for it.

There are certain exceptions (valid) to A proposition (Refer section on conversion).

(iii) Fallacy of contraposition

Example :

No dogs are bipeds.
∴ All non-bipeds are dogs.

i.e. S e P, therefore P' a S.

This is a fallacy of contraposition. The correct contraposition is 'some non-bipeds are dogs'. That is P' i S.

(iv) Amphiboly or Amphibology

This fallacy is due to ambiguous grammatical structure. A statement is said to commit the fallacy when it admits of double interpretation.

Examples :

(a) The Duke yet lives that Henry shall depose.

Ans : This may mean that either Henry will be deposed by the Duke or Henry will depose the Duke.

(b) The wolf the sheperd killed.

(c) For Sale: A Newfoundland dog. Will eat anything; very fond of children.

(d) Aristotle taught his students walking.

(e) Be Indian; buy Indian.

(f) I will wear no clothes to distinguish me from my poverty stricken fellow country men.

(v) The fallacy of account

This is due to the accent or emphasis being placed upon the improper words in a sentence. It is a fallacy arising from special stress of words in a sentence.

Examples :

1. Thou shall not bear false witness against thy neighbour.

By means of false stress either in speach or in writing, this commandment, may be interpreted in the following ways :

(a) that anything false except evidence is permitted.

(b) that false evidence may be given for him.

(c) that false witness may be given against other people (i.e. people except our neighbours).

2. Show respect to *ladies*'

Here emphasis on the last word will lead to a wrong inference that ladies alone should be respected.

3. 'Saddle the ass', the father said. The man saddled the father.

4. Men ought to be kind to *strangers.*

Section 3. Mistakes in reasoning

A. *Formal Fallacies*

These fallacies arise from violations of the rules of the syllogism. (Refer relevent sections on categorical, hypothetical and disjunctive syllogisms).

B. *Material Fallacies*

These fallacies do not result from the violation of any specific logical rules. These fallacies exist in the matter of the argument not in the form. These fallacies have their source in equivocation and presumption.

(i) FALLACIES OF EQUIVOCATION

These fallacies arise if the terms used in an argument are ambiguous and not well defined. These are called fallacies of ambiguity.

(a) *Ambiguous and Shifting Terms*

A special case of this fallacy appears in the fallacy of *ambiguous middle.*

Examples :

The end of a thing is perfection.
Death is the end of life.

∴ Death is perfection.

The 'end' in the major premise means the goal or ideal. In the minor premise it means 'the last stage.'

(b) *The fallacy of Ambiguous Major*

Examples :

Light is essential to guide our steps.
Lead is not essential to guide our steps.

∴ Lead is not light.

Here the major term 'light' is used in two different senses.

Examples :

No courageous creature flies.
The eagle is a courageous creature.

∴ The eagle does not fly.

(c) *The fallacy of Ambiguous Minor*

Examples :

No man is made of paper.
All pages are men.

∴ No pages are made of paper.

Here the minor term 'pages' is used in two different senses.

Examples :

Infantry is not a part of the human body.
Foot is infantry.

∴ Foot is not a part of the human body

(d) *Fallacy of Figure of Speech*

This fallacy occurs when we regard the same grammatical from as having the same force in every case in which it occurs. There may be ambiguity arising from, similarity of infection or prefix. We may get a wrong meaning for a word from its having a similar infection or prefix with other words of different meaning. In short, this fallacy arises from supposing words similar in form to be similar in meaning.

Examples :

1. What is seen is visible, and what is heard is audible; therefore what is desired is desirable.

Ans : Here it is wrong to argue that what is desired is desirable. 'Desirable' does not mean that which is desired but that which ought to be desired. The fact that the people desire happiness proves only that it is desired. It is no proof that it is desirable. The great logician J.S. Mill committed this fallacy.

The fallacy of figure of speech occurs through the use of paranymous terms. These terms have by no means similar meanings.

Examples :

1. Artist, artisan, artful.
2. Presume, presumption.
3. Project, projector.
4. Apprehend, apprehension.
5. Imaginary, Image.

What is 'imaginary' is unreal but an 'image' formed of wood or stone is real.

(ii) FALLACIES OF COMPOSITION AND DIVISION

Fallacies arising from the collective and distributive use of the terms are composition and division. These are due to the confusion between the collective and the distributive use of terms.

(a) *The fallacy of composition*

This fallacy arises when we argue that what is true of the parts or the individuals taken distributively is also true of them when taken collectively. (What is true of a part need not be true of a whole or group). In other words this fallacy consists in inferring from the distributive use of a term to its collective use.

Examples :

1. Neither A nor B nor C is strong enough to lift this load.

   ∴ A, B and C cannot lift this load.

2. One can live without food, for one can live without wheat or rice or meat.

3. All the angles of a triangle are less than two right angles.

   A, B and C are all the angles of this triangle.

   ∴ A, B and C are less than two right angles.

N. B : Here the world 'all' is used in the major premise in the distributive sense and in the minor premise in the collective sense.

(b) *The fallacy of Division*

1. This fallacy arises when we suppose that what is true of the whole is true of each one of its parts,. Here we proceed illogically from the collective to the distributive use of terms.

Example :

All the works of Shakespeare cannot be read in a day. The play of Hamlet is a work of Shakespeare. Therefore, Hamlet cannot be read in a day.

2. Fallacy of division resulting from the ambiguous use of the word 'all'. The word 'all' can be used distributively or collectively. In the sentence 'All crows are black', 'all' is used distributively; but in the sentence 'All these five can lift this bench', 'all' is used collectively.

Examples for the fallacy of division :

1. All these question carry 100 marks. The last questionis one of them. Therefore the last question carries 100 marks.

2. All the angles of a triangle are equal to 2 rt angles A is an angle of a triangle therefore A is equal to 2 rt angles.

(iii) FALLACY OF ACCIDENT

This has two forms (a) the direct or simple fallacy of accident. and (b) the indirect or converse fallacy of accident.

(a) *The Direct or simple fallacy of accident.*

This consists in arguing that what is true under ordinary conditions is true also under special circumstances. (This fallacy arises when we infer that what is true, normally or as a general rule, is true, absolutely and in every particular case).

Examples:

1. He who kills must be hanged.
   Therefore, all soldiers must be hanged.

2. Freedom is the birth right of man. So no one should be imprisoned.

3. Lying is wrong. So when your wife asks how you like her new dress, you should tell your true opinion.

(b) *The indirect or converse fallacy of accident*

This fallacy arises when we suppose that what is permitted under special circumstances is also permitted under all circumstances.

Example:

Alchol has restored the health of this person. Therefore, it should be taken by all.

(iv) DILEMMATIC FALLACY

This fallacy arises from the equivocal and shifting point of view present in the premises of a dilemma.

A dilemmatic argument may be fallacious on account of one or more of the following reasons.

a. In the major premise there may not be any real connection between the antecedents and the consequents. (This is one way of attacking a dilemmatic argument. This is technically known as 'taking the bull by the horns').

b. The disjunction in the minor premise may not be mutually exclusive and collectively exhaustive. (It is by pointing out that there is a third alternative, that we are said to 'escape between the horns').

c. The rule of the hypothetical syllogism may be violated either by denying the antecedent or affirming the consequent.

(For further details refer the chapter on 'Dilemma').

(ii) Fallacies of Presumption

These fallacies are the result of presumption or assumption on the part of the person making the argument. They lack an appropriate connection between the premises and the conclusion of an argument. Hence these fallacies are also called *fallacies of irrelerance.* In all these cases, the premises have nothing to do with the conclusion. It is possible (i) to assume the point to be proved in the premises of an argument (Petitio Principi); or (ii) to assume the point to be proved in a question (complex question): or (iii) to assume without warrant that a certain conclusion follows from the premises which have been stated (Non-sequitur) or (iv) that the conclusion obtained is really what is required in order to settle the question at issue (irrelevant conclusion or Ignoratio Elenchi).

1. Petitio Principi or begging the question

This fallacy is committed when we either assume the proposition we are proposing to prove, or prove it by premises which can only be proved by means of the proposition itself. In this fallacy we have only an appearance of inference. What we are really doing is repeating or resaserting what we are supposed to be proving. This may happen in three ways.

(a) *Question begging epithet*: This fallacy occurs when we assume the fact which we wish to prove or its equivalent under another name. Here the fallacy is committed in a single step of inference.

Examples :

1. Opium induces sleep because it has soporific properties. (Soporific means the same thing as inducing sleep).

2. An act is morally wrong because it is opposed to sound ethical principles.

(b) *The question may be begged by assuming a general proposition which covers the point at issue.* Such a proposition is called a question begging proposition.

This form of the fallacy is committed when we take for granted a general proposition or principle which involves the required conclusion, and which is just as much in need of proof as the conclusion itself; or when any general principle is falsely taken as self evident; or when a universal which involves the particular proposition to be proved is assumed (Here there is no formal syllogistic fallacy. The argument takes the form of a valid syllogism in Barbara. But it is materially fallacious).

Examples :

1. We should be guided by the decisions of our ancestors ; for old age is wiser than youth.

2. Knowledge of logic is not useful, because it does not teach matters of business.

3. All animals are active, and as the tortoise is an animal it is active.

(c) *A special form of petitio principi is called circle in reasoning or argument in a circle.*

This fallacy occurs when we have two propositions and use each of them to prove the truth of the other. When one of them is challenged we fall back upon the other.

Example :

1. The Vedas are infallible because they are the composition of God. But how do you know that there is God? Why, because, the Vedas declare that there is God.

N. B. : Thus the Vedas owe their infallability to God and God in His turn depends on the Vedas for proof of His existence.

2. He talks with angles; for he himself says so, and a man who talks with angels cannot lie.

3. We cannot allow unauthorised student demonstrations for unauthorised student demonstrations surely cannont be allowed.

2. **The complex question (or) the fallacy of many questions**

This is an interrogative form of petitio. This is frequently employed in the cross examination of witnesses in courts of law. It is called the leading question. It is a clever question intended to drive the party to an aukward confession. It is not really a simple question, but is based on an assumption. The assumption behind the question is the important thing.

Examples :

1. Have you stopped beating your mother ? Say Yes or No.
2. Have you given up telling lies ? Say Yer or No.
3. What did you do with the money you stole ?
4. Are you still gambling on Sundays ?

N. B : In all these cases the answer 'yes' or 'no' would imply the admission of the practice. Charles II asked the members of the Royal Society, "Why does a live fish, when placed in a bowl of water, not cause the water to overflow, whereas, a dead fish does?" The members of the society offered various explanations of the difference but no such difference in fact, exists.

Another example :

> Why are South Indians more intelligent than Nortn Indians ?
>
> Misleading questions of this kind are also called complex questions.

3. **Ignoratio Elenchi or Irrelevant conclusion**

The phrase 'ignoratio elenchi' means 'ignoring the conclusion'. It is an evasion of the point. In order to refute a thesis, one must establish the contradictory thereof. If this is ignored and something else be proved there is the fallacy of ignoratio elenchi. The English name for this fallacy is *'irrelevant conclusion'* that is establishing a conclusion which is irrelevant. In short, this fallacy consists in proving the wrong point. It is an irrelevant appeal. There are several ways in which an irrelevant conclusion may be established. Some of them have special names. They are as follows :

(i) *Argumentum ad hominem* or *Tu Quoque argument*

This is the following of abusing the man. It is a fallacy directed to the man. This is arguing about the person instead of about the proposition which he puts forward. This is very common in political controversy. Politicians generally attack the persons rather than their principles. It consists in appealing to the previous opinions and acts of the character or in showing that he himself once thought differently from what he now thinks, instead of refuting his arguments. 'Tu Quoque' means 'your are another.' 'Ad homeniem' means 'directed toward the man.' It is an irrelevant abuse of men.

Example :

> How can we listen to this man's advice now when it is well known that he has changed his opinions at least six times before ?

(ii) *Argumentum ad populum*

This consists in appealing to people's passions and prejudices rather than to their intelligence.

Example :

1. Darwin's theory of Evolution should not be accepted because its acceptance would mean that our ancestors were all apes.

2. This sewing machine must be good because it is made in USA.

(iii) *Argumentum ad misericordiam*

This is only a special case of the previous fallacy. It is a way of winning support for a measure by exciting the commiser-tion of people. It is the fallacy of illicit, irrelevant, appeal to pity.

Example:

A criminal may appeal to his judge with tears and entreaties or by bringing his wife and children into court to excite their commiseration (feelings of pity).

(iv) *Argumentum ad verecundiam*

This consists in appealing to authority instead of establishing our contention on its merits. It is a mob appeal fallacy. It is also called *ipsedixit fallacy.*

Examples :

1. The soul is immortal, because Plato says so.

2. The theory of Evolution is wrong because the Bible favours the thoery of special creation.

3. Use 'Snow White' face cream. Ten thousand people use it all over Madras. They cannot be wrong.

(v) *Argumentum ad ignorantiam*

This is an attempt to gain support from some position by dwelling upon the impossibility of proving the opposite. This consists in taking advantage of the ignorance of the hearer and making him believe that a statement has been proved while in

fact it has by no means proved. It is an irrelevant appeal to ignorance.

Examples :

1. The failure to disprove the existence of telepathy is often used as evidence for its existence.
2. Gods and spirits interfere in human affairs, because it is impossible to prove the opposite.

(vi) *Argument ad baculum*

This is appeal to the big stick. This has simply nothing to do with logic. It is an irrelevant appeal to force.

Example.

Beware of the consequences of what you propound.

(vii) *The fallacy of Objections*

This consists in saying that there are objections against some plan, statement or theory or system and hence inferring that it should be rejected.

Example.

Theory of Relativity cannot be accepted because it is not without objections.

4. The fallacy of Non sequitur or the fallacy of the consequent

The fallacy of non sequitur arises when the conclusion does not strictly follow from the premises ; that is when there is no connection between the premises and the conclusion anything can be inferred from anything else.

Example :

John must be an effective teacher because he is liked by his students.

Sometimes the fallacy of non sequitur is based on the confusion of the antecedent and the consequent of a hypothetical proposition.

Examples :

1. If the rain falls the ground is wet. Therefore, if the ground is wet rain has fallen.

2. A produces B. Therefore B Produces A.

The fallacy of non sequitur is also called *Non-Propter hoc*.

Exercises

Identify and examine the following fallacies :

1. Mohan's argument for the justification of Rama's killing of Vali must be wrong. Look how angry he gets when any one disagrees with him.

2. Extra sensory perception exists because no one has been able to prove that it does not.

3. "Sir, kindly give me pass marks. Otherwise my father will beat me and scold me and in shame, I may discontinue my studies." An argument of a student.

4. Why are college courses so irrelevant?

5. When will students begin acting responsibly?

6. Philosophers love learning because they are scholars. They are scholars because they are intellectuals. They are intellectuals because they are philosophers.

7. Salt dissolves in water. John is an old salt. Therefore John dissolves in water.

8. The faculty at my college is brilliant. The economics department is a part of the faculty at my college. Therefore the economics department at my college is brilliant.

9. The signs of the Zodiac are twelve in number. Gemini is a sign of the Zodiac. Therefore Gemini is twelve in number.

10. Can you spell backwards?

11. Logic is an indispensable aid to careful thought. For anyone who wishes to think carefully must think logically.

12. Why has the policy of open admission been a failure?

13. During the first World War a detail of soldiers were engaged in burying enemy casualities Some of the casualities protested, " But we are not dead". The work went on, however, as one of the soldiers said, "Bah! They are such liars. Why should we believe them?".

14. You must join the strike. Otherwise I will beat you.

15. There is no proof that A is false. Therefore A is True.

16. A approves statement p. Therefore statement p must be true.

17. A argues that what B says must not be accepted because B held a contrary opinion of few years before.

18. p is true because q is true. q is true because r is true. r is true because p is true.

19. Ice cream tastes sweet. So all edibles must taste sweet.

20. Food is bitter, for bitter-gourd is a food.

21. This view must be wrong for it is advocated by a hippie.

22. "Did your sales increase as a result of your misleading advertisement? Say yes or no"

23. This brand of instant coffee is good for the Indian skipper says so.

24. An Advertisement: "Have you missed the bus? Oh! we feel sorry about that! Why do not you buy Hero cycles? They never fail you."

25 A paint advertisement "Do not spoil your houses by using all sorts of paints; use always X mark paint."

26. All killers are criminals and all soldiers are killers. So, all soldiers are criminals.

27. All departures from law are punishable. All miracles are departures from law. Therefore all miracles are punishable offences.

28. Men students do not show interest in their studies. Therefore women students are bright.

29. "Lost! Lost! Lost! An umbrella by an old gentleman with a curiously carved head."

30. Pity is divine. His condition is pitiable. Therefore his condition is divine.

31. All persuasive arguments are effective. Some persuasive arguments are invalid. Therefore, some effective arguments are invalid.

32. No legal holidays fall on Sundays. All legal holidays are days when the banks are closed. Therefore, all Sundays are days when the banks are closed.

33. All good thing come in small packages. This cigarette packet must be good for it is a small package.

34. People who argue about everything are a pain in the neck. Therefore he must be a philosopher.

35. All people who work hard at their jobs deserve to be well paid; as all teachers work hard at their jobs they deserve to be well paid.

36. Since atheists do not believe in God, they must be immoral.

Chapter IX

MODERN CLASSIFICATION OF PROPOSITIONS

Sec. 1. Modern Formal Logic

Sec. 2. Modern Classification of Propositions

Section 1. Modern Formal Logic:

Formal logic, in modern times, is called symbolic or mathematical logic. Symbolic logic is an extension and continuation of traditional logic. It is complimentary to traditional logic and not contradictory to or a system altogether different from traditional logic. The spirit of logic is the same in both logics. Logic has assumed different forms. Symbolic logic is one such form.

We may tentatively say that the logic which uses symbols is symbolic logic. Of course, symbols have been used in traditional or Aristotelian logic also. For example A, E, I and O are symbols for certain kinds of categorical propositions. S and P are the symbols that are used for subject and predicate terms respectively. The word 'symbol' is used in a special sense in modern logic. Symbols are not words. They are single letters devoid of meaning. They are vacant places to be filled in by words. They are place holders. They are 'stand ins' for any term. In mathematics, we use symbols such as x, y etc. We say $(x + y)^2 = x^2 + 2xy + y^2$ Here the symbols x and y do not *mean* anything. That is, any number can take their places. Hence in mathematics they are called numerical variables. By the use of such variables things in mathematics become easy for us to understand. Using variables we are able to express a formula or principle clearly in mathematics. In the same way by the use of symbols we can express clearly without any ambiguity of language, certain logical principles and formulae and certain logical truths. Symbols serve a purpose in logic. Symbols are used in logic to make us understand and express logical principles with clarity.

Symbolic logic has two functions. They are:

(i) It helps us to determine the validity or invalidity of inferences and implications.

(ii) It helps us to determine in an a priori way the truth or falsity of propositions.

Though there are two functions they are not different.

Let us start with function (ii). Let us understand what we mean by determining a priori (analytically) the truth or falsity of propositions. To take an example (a) 'Bangalore is a beautiful city.': The truth or falsity of this proposition can be determined only with reference to empirical experience. Its truth depends on our empirical knowledge of Bangalore. Without reference to actual empir cal experience we cannot determine the truth or falsity of this proposition. But take the following proposition (b) 'Bangalore is either a beautiful city or not a beautiful city.' This proposition is a true proposition without reference to empirical knowledge. In the same way (c) 'Bangalore is both a beautiful city and not a beautiful city' is a false proposition without reference to empirical knowledge. In short, to decide the truth or falsity of proposition (b) and (c) no empirical evidence or investigation is needed. In the same way the proposition 'The accused is either innocent or guilty', is a true proposition without reference to experience. But to say that he is guilty, we require empirical evidence. That is, 'This accused is guilty' is by itself not a true proposition. Propositions which require no empirical investigations or evidences to decide their truth or falsity are *logically true or logically false propositions*. In short, empirical sentences are informative, and, therefore, depend upon experience to vouchsafe their validity. In symbolic logic we are concerned only with sentences which are non-informative. i.e. logical sentences called propositions. Hence the unit of symbolic logic is the proposition. Taking all these aspects into considerations symbolic logicians have given us a classification of propositions which is different from traditional classification of propositions. Now let us see the modern classification of propositions.

Section 2. Modern classificating propositions

(i) Traditional classification of propositions—its defects

A proposition is a constituent element of thinking or reasoning. The proposition itself has got certain constituents. These constituents are what the proposition is about. For example, the proposition, "the rose is red" is about 'the rose' and 'red'. In traditional logic (the logical doctrines of Aristotle and those who followed him) the constituents of the proposition are called the terms of the proposition.

Traditional logic has limited the constituents of a proposition to two, the subject and the predicate. Thus in the proposition 'rose is red' the constituents are 'rose' (subject) and 'red' (predicate). The S and P, in the above example, are combined by 'is'. The combining element in a proposition is called the component. Here 'is' is the component. Thus according to traditional logic, every proposition asserts a predication, that is, attributes a predicate to a subject.

According to traditional logic propositions are classified into the categorical, the hypothetical and the disjunctive.

Defects of traditional classification

(a) According to traditional logic every proposition asserts a predicate i.e. attributes a characteristic or adjective or predicate to a subject. But this is not true of every proposition. The proposition 'Rama married Sita', does not assert a predication. It asserts a relation between 'Rama' and 'Sita' and does not attribute a predicate or characteristic to a subject. In traditional logic what we do is, we artificially reduce the relation to a predication and rewrite as 'Rama is one who married Sita'. This is an unnecessary (or awkward) restatement of a relation into a predication.

(b) According to traditional logic all propositions are of the same logical form, the S – P form. This logical form limits the proposition to two constituents (S and P) and to one mode of combination (predication). But propositions need not be limited to this (S – P) form. A proposition may have any number of

constituents and these constituents may be related in various ways. Hence many logicians hold the view that the number of constituents and the mode of combination are not confined to the same logical form as assumed by the traditional logicians. This means that there is *not one* logical form. That is, the logical form of a proposition need not necessarily be in the S – P form.

Taking these points into consideration the modern logicians have classified propositions in the following manner.

(ii) Modern classifications of propositions

Modern logicians have classified propositions into
- (A) Simple propositions
- (B) Compound propositions and
- (C) General propositions.

(A) Simple propositions

Propositions which express simple facts are simple propositions.

Examples :

- (a) Thief!
- (b) The rose is red.
- (c) Sankara is a philosopher.
- (d) Rama is the husband of Sita.

Simple propositions are also called *atomic propositions* for they cannot be subdivided into further units of propositions.

Simple propositions are of four kinds. They are :
- (a) Subjectless propositions
- (b) Subject – predicate form
- (c) Class – membership propositions and
- (d) Relational propositions.

(a) Subjectless propositions

A subjectless proposition is one which does not have a logi-

cal subject. It is the most primitive form of propositions. Exclamatory propositions such as 'Thieves!' 'Fire!' 'Alas!' etc. and impersonal propositions such as 'It snows', 'It rains' etc. are examples of subjectless propositions.

(b) Subject – predicate form

Propositions which predicate an attribute to a subject are said to be in the S – P form. Example 'The rose is red.' Here 'rose' is the subject of which an attribute 'red' is predicated.

(c) Class – membership propositions

Propositions which assert that something is a member of a given class are called class – membership propositions.

Example.

Sankara is a philosopher.

A class – membership proposition is to be distinguished (i) from a proposition which states a relation of a class to a wider class and (ii) from a proposition which is of the S – P form. (i) 'All philosophers are men' is not a class – membership proposition for it states a relation of the class of philophers to a wider class 'men'. This proposition is quite different from a class – membership proposition (Sankara is a philosopher) which states the relation of an individual (Sankara) to a class (philosopher) of which the individual is a member. Thus 'Rama is a Tanjorian' is a class – membership proposition whereas 'All Tanjorians are Tamilians' is not. (ii) in the same way a class – membership proposition is different from the proposition in the S – P form. The proposition 'Sankara is wise' is a proposition in the S – P form, for it predicates the attribute 'wise' to the subject 'Sankara.' But the proposition 'Sankara is a philosopher' is not a proposition in the S – P form for here 'Sankara' is said to be a member of the class of 'philosophers.'

(d) Relational propositions

A relational proposition asserts a relation between two or more constituents. Example. Rama is the husband of Sita. In this proposition the relation is 'being the husband of.' This

relation connects 'Rama' and 'Sita ' Here 'Rama' is the *referrent* (A referrent is the term from which the relation goes) and 'Sita' is the *relatum* (A relatum is the term to which the relation goes). The proposition 'Rama is the husband of Sita' is a two-termed relation.

A relational proposition may contain any number of terms. The two-termed relations are called *dyadic* relations. Three termed relations are called *triadic*. Four termed relations are called *tetradic*. Five-termed relations are called *pentadic*. Relations having more than five terms are called *polyadic* or *multiple* relations. 'Anjaneya gave the ring to Sita' is a triadic relation. (Anjaneya, ring, Sita, the three terms are related by the relation 'give'). 'Krishna bought a house from Rama for twenty thousand rupees' is a tetradic relation. (Here the relation of 'buying' requires a *buyer, a seller, the thing bought* and the *amount* for which it is bought).

(B) Compound propositions

A compound propositions is one which is constructed out of simple propositions. It is also called a *molecular* proposition. A compound proposition is a *single* proposition. Its constituents are simple propositions. In other words, propositions which contain simple propositions as their constituents are compound propositions. Though a compound proposition contains simple propositions, it is a single proposition.

Compound propositions are broadly classified into two.

(a) conjunctive propositions and
(b) composite propositions.

(a) Conjunctive propositions

If in a proposition we combine two or more simple propositions with the conjunction 'and' it is a conjunctive proposition. Example 'Rama married Sita and Ragu married Geetha.' Here the word 'and' combines two simple propositions. A conjunctive proposition is, not an enumeration but a single proposition. The *'and'* which combines the simple propositions is used in the conjunctive and not in the enumerative sense, (e.g. Rama and Ragu

and Krishna are mortals. Here the '*and*' is used in the enumerative sense for it does not combine but only enumerates. But in the proposition 'Ragu married Geetha' 'and' 'Srinivasan became a father' the word 'and' combines two simple propositions. This conjunctive use of 'and' is called a logical conjunction). The conjunctive proposition is the simplest form of compound propositions. In English language we use 'and' between two nouns or words. But in logic we use 'and' between propositions.

(b) Composite propositions

There are certain logical conjunctions by means of which simple propositions can be combined into compound propositions. These logical conjunctions are 'if ... then', 'Either . . . or',' either or but not both'. Propositions where such logical conjunctions are used are called composite propositions.

There are three kinds of composite propositions. They are

(i) implicative propositions
(ii) alternative propositions and
(iii) disjunctive propositions.

(i) Implicative propositions

A compound proposition which combines simple propositions by means of the logical conjunction 'if . . .then' is called an implicative proposition. In it, one constituent proposition implies the other constituent proposition. Example 'if a man takes poison', he will die'. Here the proposition 'a person takes poison' implies the proposition 'he will die'. The proposition 'a man takes poison' is called the *implicans* and the proposition 'he will die' is called the *implicate*.

(ii) Alternative propositions

A compound proposition which combines two or more simple propositions by means of the logical conjunction 'either ... or' is called an alternative proposition. Example: Either he is industrious or he is intelligent'. Two simple propositions 'he is industrious.' 'he is intelligent' are combined here by the phrase 'either . . . or.' The components of the alternative propositions are called *alternants*.

In this form of the proposition what we want to assert *is not that both alternants cannot be true but* at least one of them must be true In an alternative proposition we do not exclude the possibility of both alternants being true. That is, by means of an alternative proposition we want to assert that at least one of the alternants is true. The alternative proposition is also called inclusive disjunction.

(iii] Disjunctive propositions

A compound proposition which combines two or more simple proposition by means of the logical conjunction, 'either or but not both' is called a disjunctive proposition. It is also called exclusive disjunction. Example: 'He is either a bachelor or a married man but not both'. Here two simple propositions 'he is a bachelor', 'he is married', are combined on the basis of mutual exclusion or logical disjunction. The components of a disjunctive proposition are called disjuncts. The disjuncts are combined on the principle of mutual exclusion. That is, what we want to assert in a disjunctive proposition is that both alternants cannot be true at the same time and if one of them is true, the other must be false.

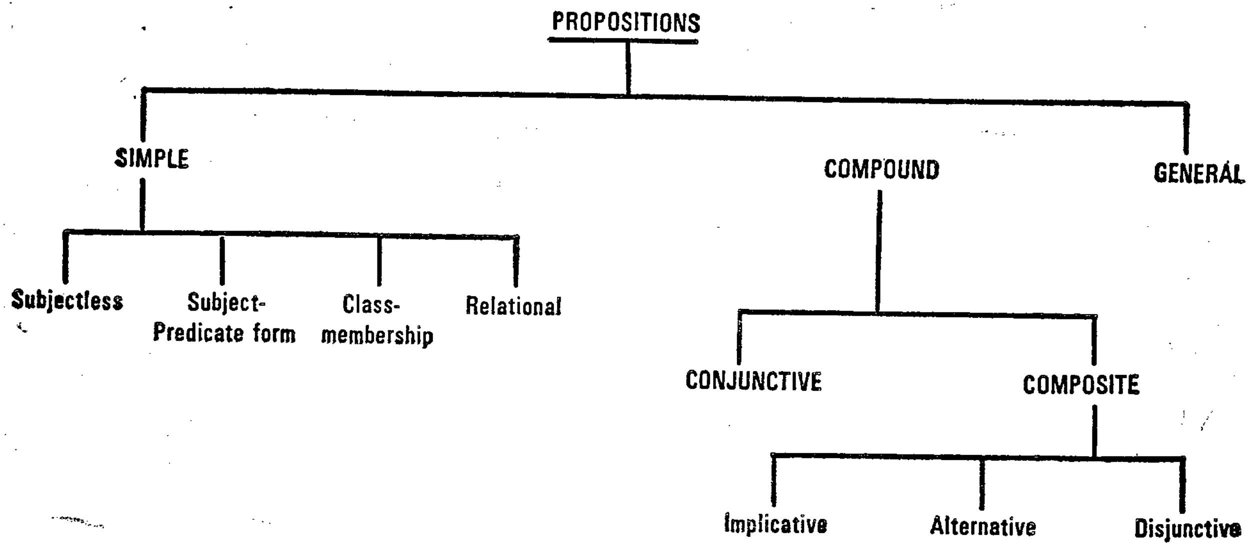
C. General proposition

Propositions. which assert that one class is (entirely or partially) included or excluded from another class are called general propositions. Examples: 'All Tanjorians are Tamilians.' 'Some men are not teachers', All general propositions are qualified statements which have either 'all' or 'some.'

General propositions differ from class membership propositions. A class membership states a relation of an invididual to the class of which it is a member (e.g. Rama is a Tamilian). But a general proposition asserts relation of one class to another (e.g. All Tamilians are Indians). Further a class membership proposition is a simple proposition. But a general proposition is not a simple proposition. Nor is it a combination of simple propositions like compound propositions. It is a proposition which asserts a relation of inclusion or exclusion (total or partial) between classes.

We may conveniently represent the modern classification of propositions in the following table:

MODERN CLASSIFICATION OF PROPOSITIONS



NOTE : In the traditional classification some kinds of propositions are completely ignored (i) The traditional classification did not make a clear distinction between (a) Simple and Compound propositions (b) Simple and General propositions (c) Conjunctive and Composite propositions (d) Propositions of the S–P form and relational propositions and (e) Alternative and disjunctive propositions. (ii). The traditional logicians called the 'implicative' propositions the hypothetical, and the 'alternative' as well as the 'disjunctive' proposition as disjunctive.

Exercises

Point out to what type of modern classification the following propositions belong. Explain:

1. Gujarat is a state.
2. Sentences are either right or wrong but not both.
3. Tensing conquered the Everest.
4. All spiders are eight-legged creatures.
5. This drama is not a tragedy.
6. If Morarji Desai visited Washington President Carter would be pleased.
7. Lava is twin of Kucha.
8. A few women are not mothers.
9. Caitra eats either by day or by night.
10. Wherever there is smoke, there is fire.
11. Rama was killed and John was injured in that road accident
12. Thief! Thief!
13. A is equal to B.
14. The principal must be firm, yet he should not be uncompromising.
15. If Rama is a professor, then he is over thirty.
16. It will rain and snow.
17. No wasps are spiders.
18. Our ends may be desirable, but our means are questionable
19. A baby is either a boy or girl.
20. Vyasa is the author of Mahabaratha.

21. Raju is either a knave or a slave.

22. The signal light is either red or green but not both.

23. All Men are falliable.

24. Newton is a physicist.

25. If war is declared then prices go up.

26. Tagore is either a poet or a philosopher.

27. Logic is worthy of study.

28. Men must work and women must sweep,

29. Drona is the teacher of Arjuna.

30. You must either pay the fine or leave the college.

31. Jack fell down and broke his crown.

32. Many adults are married.

33. Devadatta is in the house.

34. Either war must be eliminated or the human race must be eliminated.

35. Rama is a teacher.

36. If this report is true, then you should be punished.

37. He is either a friend or a foe but not both.

38. If you lend me a rupee, I will return it next month.

39. A given number is either odd or even but not both.

40. God exists.

41. If the sun is shining, them it is day.

42. Neither wealth nor honour can make you happy.

43. Radhakrishnan is a philosopher.

44. Rama killed Ravana.

45. That dog is a fox terrier.

46. If the dry weather continues, the harvest will be bad.

47. The brave gave their lives and the cowards made money.

48. Either corruption is to be eliminated or revolution cannot be avoided.

49. What fools these mortals be?

50. Neil Armstrong walked on the moon.

Questions

I. What, according to modern classification, is a compound proposition? Explain the different kinds of compound propositions.

II. Explain and illustrate the different types of simple propositions recognised in modern logic.

III. Explain the following through examples :

(a) Alternative proposition.

(b) Conjuctive proposition.

(c) Implicative proposition.

IV. Distinguish between

(a) Simple and compound propositions.

(b) Alternative and disjunctive propositions.

(c) Class membership and general propositions.

V. Represent the modern classification of propositions by means of a table. What, according to you, are the defects of the traditional classification? Show how the modern classification is an improvement over the traditional classification.

Chapter X

PROPOSITIONAL CALCULUS

Sec. 1. Introduction.

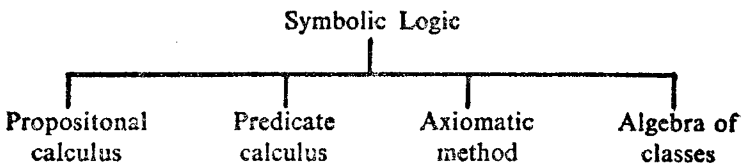
Sec. 2. Symbolic representation of propositions.

Sec. 3. Construction of Basic Truth Tables.

Sec. 4. Truth tables as decision procedure.

Section 1. Introduction

Symbolic logic can be classified under the following heads. It includes 1. propositional calculus, 2. predicate calculus, 3. axiomatic method and 4. algebra of classes.



Propositional calculus : deals with arguments based on simple and compound propositions. It treats propositions as basic unanalysed units. Propositional calculus is known by various names in symbolic logic. It is known as *sentential calculus, truth functional logic* or *algebra of logic* or *propositional logic.*

Predicate calculus deals with arguments based on simple and general propositions. It is also called *predicate logic.*

Axiomatic method is a theory used to justify or prove the independence, consistency and completness of any system of logic.

Algebra of classes The Boolean algebra of classes and Cantor's set theory may be considered as algebra of classes.

Without entering into details we may simply note here that there are certain words which occur in all sciences, irrespective of their subject matter. What are these words? They are, 'and'

'not', 'if then', 'either or', 'all', 'some'. Though these words are used by several sciences, the implication of these words are not studied by any one of them. Logic deals with these words and makes a special study of these words and the implications of propositions and arguments using these words. Since logic as a science studies these words which are used in every other science, logic is called *the science of sciences*. As an illustration let us take the word 'and' and explain how it is used in logic. In language, the word, 'and' is used in the sense of conjunction between two *related* sentences. The word 'and' is used in language between sentences which come in a *particular order*. But in logic the word 'and' is used to stand for conjunction between *any* two sentences irrespective of their meaningful relation. In English language we can use the word 'and' between (two) connected sentences such as 'John is a christian' and 'he goes to church'. But in logic we can use the word 'and' to connect any two unconnected sentences also such as 'John is a christian' *and* 'Tirupati is a big town.' In the same way we can say that 'if John is a christian then Tirupati is a big town'. Here 'if . . .then' simply connects two propositions. We can also have another proposition by saying 'Either John is a christian or Tirupati is a big town' ? The words 'and' 'either or', 'if . . .then' connect two propositions which are otherwise unconnected. In using these words of connection no fixity of order or meaning is implied in logic. *Propositional calculus* studies the logical implication of the words 'and' 'if then' 'either or' as used in arguments, whereas *predicate calculus* studies the logical implication of the words 'all' and 'some' as used in arguments. Propositional logic and predicate logic are techniques not theories. But axiomatic method is a theory of these two branches of logic but it is not a technique. With these introductory remarks let us see the various symbols used in symbolic logic.

Section 2. Symbolic representation of propositions

The aim of symbolic logic is to set up an artificial symbolic language with which we can simplify complicated logical arguments. Tamil, Sanskrit, English, French etc are languages which use a kind of symbolic system. But the symbols of these languages are not universal. The aim of symbolic logic is to set up a sort of universal language or script by using artificial symbolic language. Ordinary language uses sentences and words which at

times have an emotional appeal. It also uses words and sentences in an ambiguous sense. To avoid these difficulties we use artificial symbolic language. The language we use in symbolic logic is simpler than any natural language, in that it does not contain overtones. Symbolic language is a notational shorthand.

In symbolic logic we denote *simple propositions* with letters such as p, q, r,....... The simple propositions are connected by words like 'and'. 'if then'. 'either or'. Letters p, q, r are called *variables*. Words like 'and', 'if then', 'either or', 'not' are called *constants*. Variables are place holders. Their meanings vary from propositions to propositions. Any concrete proposition may substitute or replace the variables. Constants are those which have fixed meanings.

Let us explain variables and constants by taking an example from arithmetic.

	constant		constant	
	↑		↓	
x	+	y	=	z
↓		↓		
variable		variable.		

x and y can stand for any number. Therefore, they are called *variables*. + and = are constants. In the same way in the propositions.

(i)	p	and	q
	|	|	|
	variable	constant	variable

(ii)	If	p	then	q
	|	|	|	|
	constant	variable	constant	variable

(iii)

Either	p	or	q
|	|	|	|
constant	variable	constant	variable

(iv)

If and only if	p	then	q
|	|	|	|
constant	variable	constant	variable

(v)

Either	p	or	q	but not both
|	|	|	|	|
constant	variable	constant	variable	constant

(vi)

not	p
|	|
constant	variable

p and q are variables because in all these above propositions we can substitute any propositions for p and q. Consider the following :

(i) I am a man *and* you are a boy.

(ii) *If* I am a man, *then* you are a boy.

(iii) *Either* I am a man *or* you are a boy.

(iv) *If and only if* I am a man *then* you are a boy.

(v) *Either* I am a man *or* you are a boy *but not both.*

(vi) I am *not* a man.

In all these examples (i) to (vi) 'and', 'if then', 'either or', 'if and only if ', 'either or but not both', 'not' are not changing. They are called logical constants.

To denote variables we use small letters p, q, r, ... ' p, q, r, ...' are symbols for variables. The following symbols are used for constants in symbolic logic.

for 'and', we use the symbol '.'

for 'not', we use the symbol '—' (usually the symbol '∼' is used).

for 'either...or', we use the symbol 'V'

for 'either...or and but not both',
we use the symbol '$\wedge$'

for 'if...then', we use the symbol '$\supset$'

for 'if and only if'
we use the symbol '$\equiv$'

We can represent *propositions* (i) to (vi) above *in the following* as symbolic formula.

(i) $p \cdot q$

(ii) $p \supset q$

(iii) $p \vee q$

(iv) $p \equiv q$

(v) $p \wedge q$

(vi) $-p$

The formulae '$p \cdot q$', '$p \vee q$' etc. (i.e.) (i) to (vi) above) are called *propositional forms*. A propositional form is different from a *proposition*. This is the case with traditional logic also. The propositional form of a universal affirmative proposition is 'All S is P '. Any word can be substituted for S and P. For instances 'All S is P' may be substituted by the proposition 'All roses are red' or by 'All men are mortal'. 'All S is P', is only a propositional form. 'All roses are red', is a proposition. Same is the case with symbolic logic. '$p \cdot q$', '$p \vee q$', '$p \wedge q$', '$-p$', '$p \equiv q$', '$p \supset p$' are propositional forms or *propositional functions*. In order to make them propositions we must make appropriate substitutions. In symbolic logic this process of substitution is carried out by the use of the capital letters representing propositions.

If *John is a christian,* then *Tirupati is a big town.*
J T

For the proposition 'John is a christian' we use a prominent letter of the proposition 'J' in capital. For the proposition 'Tirupati is a big town' we use a prominent letter of the proposition 'T' in capital. Thus

'If J then T' is a proposition.

'If p then q' is a propositional form.

The same is the case with *argument* and *argument forms.*

Let us take an example:

If Rama takes poison, he will die.

Rama has taken poison.

$\therefore$ *He will die.*

If we represent this as

R ⊃ H (R = Rama takes poison
H = He will die)

R

$\therefore$ H

then it is an argument.

Instead of this if we represent this as

p ⊃ q

p

$\therefore$ q

then it is an argument form.

So far we have seen wnat is meant by 'variables', 'constants', 'propositions', propositional form', 'arguments' and 'argument forms' in symbolic logic.

Let us know *some more technical terms now.*

' . ', ' — ', ' V ', ' Λ ', ' ⊃ ', ' ≡ ' are *constants.*

Constants are either *connectives* or *modifiers.*

All constants except '—' are connectives. Since '—' modifies the value of the original proposition it is called a modifier. Modifiers and connectives are called constants. They are also

called *operators* or *functors*. A functor is an expression which forms a proposition from propositions. '—' is a *monadic functor*. All other constants are called *dyadic functors.*

Dyadic functors take two propositions to form propositional functions. A *monadic functor* takes only one proposition to form a propositional function.

Negation (—) is a monadic functor.

Rama is a student. (p)

Rama is not a student (—p)

Only one proposition is required for forming a negative function of a proposition.

Conjunction (and all other constants except negation) is a dyadic functor.

Rama is a student. (p)

Sita is a girl (q)

Rama is a student and *Sita is a girl.*

p q

To be a functor 'and' requires two propositions. So it is a dyadic functor. Similarly 'if then', 'either or' are dyadic functors. In short, the functors 'and', 'either or' etc. require flanking propositions or variables to make any sense.

__________ and __________

1 2

either __________ or __________

1 2

Therefore, they are dyadic connectives. But the functor 'not' is a one place connective. It operates on one proposition only. Therefore, 'not' is a monadic connective.

Let us take an example to clarify what we have studied so far.

It is raining.
The ground is wet } These are simple propositions.

These two propositions can be connected to form different *functions of propositions.*

If it is raining, then the ground is wet.
Either it is raining or the ground is wet.
It is not raining.
The ground is not wet. } All these are propositional functions.

In the example

If. it is raining then *the ground is wet.*

	proposition	functor	proposition

The whole statement is a propositional function.

Expression such as 'p . q, 'p ∨ q', 'p ⊃ q', '—p', 'p ≡ q', 'p ∧ q' contain variables and constants. Any expression with variables and constants is called a *formula.* All the above expressions are formulae.

Logicians talk about two kinds of formulae. They are

(i) Well-formed formula.

(ii) Ill-formed formula

The order in which we write a formula makes it well-formed or ill-formed. Thus

'—p', 'p . q', 'p ⊃ q'

are well-formed formulae. The abbreviation for well-formed is wff.

'p—', '. pq', 'pq ⊃ '

are ill-formed formulae.

Let us summarize what we have learnt so far in the form of a table. This table may be called the '*symbolic table*'.

THE SYMBOLISM OF THE PROPOSITIONAL CALCULUS

Constants (or) Connectives	symbol for constant (or) Connective	Name of Symbol	Name of Proposition	Propositional Form	Example
NOT	—	Dash	Negation	—P	It is *not* raining.
AND	.	Dot	Conjunction	p . q	It is raining *and* the ground is wet.
If then	⊃	Horse Shoe	Implication (or) Conditional	p ⊃ q	*If* it is raining *then* the ground is wet.
Either or . . . or both	∨	Vel/ Wedge	Alternation (Inclusive disjunction)	p ∨ q	*Either* it is raining *or* the ground is wet.
Either or . . . but not both	∧	inverted Vel	Exclusive disjunction	p ∧ q	*Either* it is raining *or* the ground is wet *but not both*.
If, and only if, then	≡	triple bar	Material Equivalence (or) bi-conditional	p≡q	*If and only* if it is raining, then the ground is wet. Instead of writing the example this way some logicians use the term '*iff*' to denote 'if and only if'. They write as: '*iff* it is raining, then the ground is wet.'

Section 3. Construction of Basic Truth Tables

In symbolic logic, every proposition has two values, truth and falsity. Sentences with no truth value are of no concern for the logician. We will call the truth and falsity *truth-values*. The truth value of a proposition is thus its truth or falsity. If a statement is true, its truth value is T; and if it is false, its truth value is F. Every proposition (whether we know its truth value or not) must be either T or F.

A *truth table* is a device that shows how the truth values of statements depend on the connectives used and on the truth values of the component simple propositions. In short, a truth table is a device giving all possible truth values of a formula.

The truth tables for negation, conjunction, disjunction (both inclusive and exclusive) implication and material equivalence are called *basic truth tables*. Basic truth tables are constructed for one variable and for two variables. We may construct truth tables for more than two variables, i.e. three, four or five variables. Construction of truth tables for more than two variables is called non-basic truth table. But this is beyond the scope of the present book.

Basic truth Tables

1. The *truth table for negation* is the truth table for one variable:

P	—p	It means
T	F	if p is T, — q is F
F	T	if p is F, — q is T

To denote negation in addition to 'not' we use the expressions 'it is false', 'it is not the case that'.

2. *Truth table for conjunction (and):*

p	q	p.q	It means
T	T	T	if p is T, q is T, conjunction is T
T	F	F	if p is T, q is F, conjunction is F
F	T	F	if p is F, q is T, conjunction is F
F	F	F	if p is F, q is F, conjunction is F

3. *Truth table for disjunction (inclusive):*

p	q	p∨q	It means
T	T	T	if p is T, q is T disjunction is T
T	F	T	if p is T, q is F disjunction is T
F	T	T	if p is F, q is T disjunction is T
F	F	F	if p is F, q is F disjunction is F

4 *Truth table for exclusive disjunction:*

p	q	p∧p	It means
T	T	F	if p is T, q is T Exclusive disjunction is F
T	F	T	if p is T, q is F Exclusive disjunction is T
F	T	T	if p is F, q is T Exculsive disjunction is T
F	F	F	if p is F, q is F Exclusive disjunction is F

5. *Truth Table for material equivalence :*

p	q	p≡q	It means
T	T	T	if p is T, q is T equivalence is T
T	F	F	if p is T, q is F equivalence is F
F	T	F	if p is F, q is T equivalence is F
F	F	T	if p is F, q is F equivalence is T

6. *Truth Table for Implication:*

p	q	p⊃q	It means
T	T	T	if p is T, q is T implication is T
T	F	F	if p is T, q is F implication is F
F	T	T	if p is F, q is T implication is T
F	F	T	if p is F, q is F implication is T

In all the above cases (1 to 6) the letters p, q represent variables. Every proposition has two values (T and F). In the tables mentioned above we deal with two variables (two propositions p, q) and hence they must possess $2 \times 2 = 4$ truth values. In order to exhaust all possible combinations of Ts and Fs we write two Ts followed by two Fs in the first column of the first variable. In the second column of the second variable one F is written after one T alternately.

To sum up :

In *conjunction*

only when both variables are T then p . q is T; otherwise p . q is F.

In *inclusive disjunction*

only when both variables are F the disjunction, p ∨ q, is F; otherwise it is T.

In *exclusive disjunction*

only when both variables are T or
only when both variables are F
$p \wedge q$ is F; otherwise it is T.

In *material equivalence*

only when both the variables are T or
only when both the variables are F
then $p \equiv q$ is T; otherwise it is F.

In *implication*

only if it is T → F it is F;
otherwise it is T.

Section 4. Truth tables as decision procedure

Decision procedure is the method of testing the validity of arguments. In the truth table decision procedure we use the basic truth tables. By this procedure we may test the validity of the formula or the formulae or the validity of argument forms.

Truth table Decision procedure for formulae :

Let us use the basic truth tables to determine the validity of the following formula.

$$[\,(p \supset q) \cdot p\,] \supset -q$$

This is a formula. Here there are two kinds of brackets. (   ) is called ordinary brackets. [   ] is called square brackets. Sometimes a third type of brackets called double brackets {   } is also used. Brackets decide the scope of constants. In the above example, ordinary brackets include ($p \supset q$) and not others. The square bracket covers ($p \supset q$) and p. The main constant of this example is the symbol of implication outside the square bracket. The symbol of negation belongs to q and it governs only q and so it is not the main constant.

The term '*table value*' is used to indicate the value of the main constant.

Now let us work it out.

[(p	⊃	q)	.	p]	⊃	—	q
↓	*	↓	√	*	.	√	
T	T	T	T	T	F	F	T
T	F	F	F	T	T	T	F
F	T	T	F	F	T	F	T
F	T	F	F	F	T	T	F
↓ Ist Step	↓ Truth value of ⊃ (2nd step)	↓ Ist Step		↓ Ist Step		↓ Truth value of negation (4th step)	↓ Ist Step

Truth value of 'and' (3rd step) is arrived at by taking into consideration the truth value of the table indicated by*

Truth value of ⊃ (the main constant) is arrived at by taking into consideration truth value of the tables indicated by √ (last step)

The constant whose truth value is determined in the last stage in the truth table is the main constant.

In the above example, we have a mixture of Ts and Fs under the main constant. When the table value is a mixture of Ts and Fs, the formula is said to be *contingent*.

In some cases the table value may be all Ts. Then the formula is said to be *tautology*. Tautology is also called logical truth.

In other cases the table value may be all Fs. Then the formula is said to be *contradictory*. In the case of arguments tautology only is valid and the others are invalid.

The truth value of the formula

$$[\,(P \supset q) \cdot P\,] \supset -q$$

is, therefore. *contingent*.

Decision procedure for arguments.

Let us test the validity of the following argument.

If Stalin is a Russian, then Tagore is an Indian.
Tagore is an Indian
∴ Stalin is a Russian

Let us symbolize the argument first.

(Let us use S = Stalin is a Russian.
T = Tagore is a Indian.)

Then the symbolic form of the argument is

If S, then T
T
∴ S

We have to put it in the formula. The corresponding formula of the argument is

$[(S \supset T) . T] \supset S$

For conjoining the premises we use the symbol 'and'.

For conclusion which is the implication of the argument we use the symbol $\supset$.

Then we apply the basic truth tables to the above formula.

[(S	⊃	T	.	T)]	⊃	S
T	T	T	T	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	T	F	F	F	T	F

This formula is *contingent*. Therefore, the argument is invalid.

Exercises

Before doing the exercises remember the following :

Basic Truth Tables

p	q	p . q	p ∨ q	p ∧ q	p ≡ q	p ⊃ q
T	T	T	T	F	T	T
T	F	F	T	T	F	F
F	T	F	T	T	F	T
F	F	F	F	F	T	T

(It is advisable to use composition (i.e. widely ruled) note books to do exercises in symbolic logic based on truth tables).

I. Identify the constants and propositions in the following:

1. Either A wins his first game and C wins his first game or D wins his first game.
2. A wins his first game and either C wins his first game or D wins his first game.
3. If A win his first game then both C and D win their first games.
4. If A wins his first game then either C or D wins his first game.
5. If A does not win his first game then it is not the case that either C or D wins his first game.
6. Life is short and art is long.
7. Heidegger is a physicist or Heisenberg is a physicist.
8. If the book is blue, then it is coloured.
9. If it rains today then Ram repairs his umbrella.
10. Either I am Caeser or I am nothing.
11. Shanker will get his Ph.D., only if his thesis is approved.
12. If and only if it rains then the match will not be played.

II. Given that p has value T and q has value F determine the value of the following :

1. $(p.q) \supset q$
2. $(p \supset q) . -q$
3. $p \supset [(p \vee q) \vee q]$
4. $p \vee -q$
5. $p . -q$
6. $p \supset -q$

7. $p \equiv -q$

8. $-(p . -q)$

9. $(p.q) \vee (q.p)$

10. $(p \vee q) \supset (q > p)$

III. Determine whether the following formula is a tautology or not :

1. $(p \supset q) \supset (-q \supset -p)$

2. $[(p \vee q) . -p] \supset q$

3. $(p.q) \equiv (q.p)$

4. $(p \vee q) \equiv p$

5. $p \supset (p \vee q)$

6. $(p.q) \supset (p \vee q)$

7. $(-p \supset p) . (p \supset -p) .$

8. $(p \vee q) \supset (p.q)$

9. $p \quad (q \supset p) .$

10. $-[p. -(p \vee q)] \supset [q. -(q \vee p)]$

IV. Get as many functions as possible from the following two propositions using the various constants :

1. This umbrella is black.

2. This pen is red

V. If A and B are true statements and X and Y are false statements which of the following compound statements are true ?

1. $X \supset (X \supset Y)$

2. $(X \supset X) \supset Y$

3. $(A \supset X) \supset Y$

4. $(X \supset A) \supset Y$

5. $A \supset (X \supset B)$

6. $A \supset (B \supset Y)$

7. $(X \supset A) \supset (B \supset Y)$

8. $(A \supset X) \supset (Y \supset B)$

9. $(A \supset B) \supset (-A \supset -B)$

10. $(X \supset Y) \supset (-X \supset -Y)$

11. $[(A.X) \supset Y] \supset (A \supset Y)$

12. $[X \supset (A \supset Y)] \supset [(X \supset A) \supset Y]$

VI. Use truth tables to determine the validity or invalidity of each of the following argument forms :

1. $p . q$
   $\therefore p$

2. p
   $\therefore p . q$

3. $p \vee q$
   $\therefore p$

4. p
   $\therefore p \vee q$

5. p
   $\therefore p \supset q$

6. p
   $\therefore q \supset p$

7. $p \supset q$
   $\therefore (-q \supset -p)$

8. $p \equiv q$
   $\therefore -p \supset -q$

9. $p \vee q$
   p
   $\therefore -p$

10. p
    q
    $\therefore p \cdot q$

11. $p \supset q$
    $q \supset p$
    $\therefore p \vee q$

12. $p \supset q$
    $p \vee q$
    $\therefore q$

VII. Use truth tables to characterise the following statement forms as tautologous, contradictory or contingent :

1. $P \supset -p$
2. $(p \supset -p) \cdot (-p \supset p)$
3. $p \supset (p \supset p)$.
4. $(p \supset p) \supset p$
5. $p \supset (p \cdot q)$
6. $(p \cdot q) \supset p$
7. $(-p \cdot q) \cdot (q \supset p)$.
8. $[(p \supset q) \supset q] \supset q$
9. $[(p \supset q) \supset p] \supset p$

VIII. Use truth tables to determine the validity or invalidity of each of the following arguments:

1. If a person reads 'The Hindu' then he is well-informed. This person is well-informed. Therefore this person reads 'The Hindu'.

2. If Ram understands logic, then he enjoys this sort of problem. Ram does not enjoy this sort of problem.

3. Mary has a little lamb or a big bear, Mary does not have a big bear. Therefore, she has a little lamb.

4. If Raju uses "Smiles" tooth paste, then he has fewer cavities. Therefore, if Ram has fewer cavities then he uses "Smiles" toothpaste.

5. Either apples are ripe or wholesome and not both. This apple is ripe. Therefore, it is not wholesome.

6. If and only if the bus breaks down, then we will not go on an excursion ; the bus breaks down.

IX. Suppose p = 'I eat fish' and q = 'I am strong', translate the following propositions into words:

1. $p \cdot q$
2. $-P$
3. $-(p v q)$
4. $-p v q$
5. $-(-p)$
6. $p \cdot -q$

X. Let p represent the proposition 'Rama is active' and 'q' represent the proposition 'Krishna is active'. Translate each of the following propositions into symbolic formula:

1. Both Rama and Krishna are active.
2. It is not the case that both Rama and Krishna are active.
3. Rama is active or Krishna is not active.
4. Rama is active and Krishna is not active.
5. Either Rama is active or Krishna is active, but not both.

XI. Find out the truth value of the following:

1. $-p \cdot (-q)$
2. $-(p \cdot q)$
3. $-p \vee (-q)$
4. $p \cdot (p \vee q)$
5. $r \cdot (-s)$
6. $s \vee (-s)$
7. $(r \cdot s) \vee (-s)$
8. $(p \vee q) \vee (q \cdot p)$
9. $[p \cdot (q \vee -p)] \vee (p \cdot -q)$
10. $(p \cdot q) \cdot p$

Questions

1. Distinguish between:
   (a) the logical and linguistic uses of 'and'.
   (b) variables and constants.
   (c) proposition and propositional form.
   (d) argument and argument form.
   (e) connectives and modifiers.
   (f) monadic and dyadic functors.
   (g) wff and ill-formed formula.
   (h) Tautology and contingent truths.
2. State and explain the basic truth tables.

Chapter XI

RELATIONS AND RELATIONAL ARGUMENTS

Sec. 1. Relations.

Sec. 2. Relational arguments.

Section 1. Relations

Relations are more easily illustrated than defined. 'Greater than' 'the father of', 'equal to' etc., are examples of relations which are in common use. Every relation has a sense. By sense we mean the direction in which the relation goes. Example, 'Ragu is the husband of Geetha.' Here the relation 'husband of' goes from the male to the female. The term from which the relation goes is the *referent.* The term to which the relation goes is the *relatum.* Thus, in the above example, Ragu is the referent 'Geetha is the relatum' and 'being the husband of' is the *relation.*

Relations are distinguished according to the number of terms into dyadic, triadic, tetradic, pentadic and polyadic.

Relations have certain logical (formal) characteristics (properties). They are: 1. Symmetry 2. Transitivity (Transtiveness) 3. Correlation and 4. Connexity. On the basis of these formal properties relations (i.e. dyadic relations) are classified as follows:

Classification of relations on the basis of their logical (formal) properties.

1. Symmetry is a logical property on the basis of which relations are classified into (a) symmetrical (b) asymmetrical and (c) non-symmetrical.

(a) *Symmetrical relations.* Example: 'Suresh is the brother of Ravi.' So 'Ravi is the brother of Suresh.' 'The brother of' is a symmetrical relation because it is the same as its converse. Other examples of symmetrical relations :—cousin of, sister of (if used

between sisters), as old as, as young as, as tall as, as short as, married to, spouse of, equal to, unequal to, different from, identical with, similar.

(b) *Asymmetrical relations*: Example: 'Usha is the daughter of Savithiri.' Here Savithiri does not stand in the same relation to Usha as Usha stands to Savithiri. (For Savithiri is the mother of Usha). Hence the relation 'being the daughter of' is said to be an asymmetrical relation. Asymmetrical relation is one which is incompatible with its converse. (Other examples : 'Kannan is the brother of Geetha'; Jayashree is the sister of Mohan'). Other examples of asymmetrical relations—Father of, mother of, wife of, husband of, before, after, greater than, brighter than, taller than, smaller than, parent of, child of, servant of, ancestor of.

(c) *Non-symmetrical relations :* Example: 'A loves B.' But B may love A or B may not love A. Here the relation 'loves' is a non-symmetrical relation. Relations which are neither symmetrical nor asymmetrical are non-symmetrical. Other examples of non-symmetrical relations, 'hates', 'prefers', 'owes', 'gives.'

2. **Transitivity** is a logical property on the basis of which relations are classified into (a) transitive, (b) intransitive and (c) non-transitive.

(a) *Transitive relations* : Example. If A is the ancestor of B and B is the ancestor of C, then A is the ancestor of C. Here the relation 'the ancestor of' is transitive. Other examples of transitive relations : older than, younger than, east of, infront of, equal to, precedes, inclusive of, greater than.

(b) *Intransitive relations :* Example: If A is next to B and B is next to C then A is not next to C. Here the relations 'next to' is intransitive. Other examples of intransitive relations—father of, mother of, spouse of, married to.

(c) *Non-transitive relations :* Relations that are sometimes transitive and sometimes intransitive are non-transitive relations. Examples: If A is the friend of B and B is the friend of C, then A may be the friend of C or may not be the friend of C. Other examples of non-transitive relations: different from, cheat ng, lover of, employer of, cousin of, admires, trusts, hates.

The properties of symmetry and transitivity are independent of one another. Hence we may have any of the following nine types of relations. (i) Transitive symmetrical (ii) transitive asymmetrical (iii) transitive non-symmetrical (iv) intransitive symmetrical (v) intransitive asymmetrical (vi) intransitive non-symmetrical (vii) non-transitive symmetrical (viii) non-transitive asymmetrical and (ix) non-transitive non-symmetrical.

3. **Correlation** is a logical property based on the number of objects to which a given term (referent or relatum) may be connected by the given relation. On the basis of correlation, relations are classified into (a) many-many, (b) many-one (c) one-many and (d) one-one.

(a) *Many-many*: The relation 'friend of' is a many-many relation. If A is the friend of B, other men besides A may stand in such a relation to B and other men besides B may stand in that relation to A. Another example A loves B.

(b) *Many-one* : The relation 'son of' is a many-one relation. If A is a son of B, other individuals (C, D) besides A may stand in this relation to B. But B is the only one to whom A may stand in this relation.

(c) *One-many* : The relation 'father of' is a one-many relation. Thus in B is the father of A, B may stand in this relation to other individuals besides A but only one individual can stand in this relation to A.

(d) *One-one* : The relation 'eldest son of' is a one-one relation. Thus in 'Rama is the eldest son of Dasaratha' only one individual Rama may stand in this relation to Dasaratha and there is only one individual Dasaratha, to whom Rama may stand in this relation.

Section 2. Relational arguments

The validity of logical inferences depend on the nature (properties) of the relations. We may see some of them briefly.

(a) Immediate inference by conversion depends on the property of symmetry.

(i) In cases where simple conversion of proposition A is not possible the relation is not symmetrical. Example: From 'all monkeys are animals' we cannot infer 'all animals are monkeys' for the relation between 'monkeys' and 'animals' is not symmetrical.

(ii) In cases where simple conversion is possible (propositions E and I) the relation is symmetrical. Example: From 'no men are angles' we can infer that 'no angels are men' for the relation is symmetrical (exclusion).

(b) Categorical syllogisms, hypothetical syllogisms and relational (a fortiori) arguments depend on the property of transitivity.

(i) Categorical syllogisms where all the three propositions are general propositions the relations are transitive. Examples: All men are mortal, all ministers are men, therefore all ministers are mortal.

(ii) In pure hypothetical syllogisms (where all the three propositions are hypothetical) the relations are transitive.

Example:

If A is B, C is D.
If C is D, E is F.

∴ If A is B, E is F.

Here the relation of the implication is transitive.

(iii) The validity of sorities and relational (a fortiori) arguments depend on the property of transitivity.

Example:

A is the ancestor of B; B is the ancestor of C. Hence A is the ancestor of C.

Thus the validity of immediate inference depends on the property of symmetry; the validity of mediate inferences depends on the property of transitivity.

Exercises

I. *State whether the relation in each of the following is symmetrical, asymmetrical, non-symmetrical, transitive, intransitive, non-transitive, one-one, many-many, one-many or many-one.*

1. Latha is the sister of Usha.
2. Latha is the sister of Suresh.
3. Kannan is the cousin of Ramanan.
4. Mary loves John.
5. Madras is east of Kancheepuram.
6. Rama is the son of Dasaratha.
7. TNG is the friend of GVS.
8. He is the shortest man in the army.
9. Brutus killed Caesar.
10. Ragu is as tall as Seenu.
11. Arjuna had the same parents of Bhima.
12. This patch of colour is brighter than that.
13. A is greater than B.
14. The anthropoid ape is the ancestor of man.
15. God is the protector of man.
16. Govinda is debtor to Gopalan.
17. Rama is the husband of Sita.
18. Appu is the grandson of Kuppu.
19. Nehru was the contemporary of Gandhi.
20. Raju is the employer of Ramu.
21. Romeo is the lover of Juliet

22. In hockey Pakistan was defeated by India.

23. Radha is a next door neighbour of Geetha.

24. Lava is twin of Kucha.

II. *Test the following arguments :*

1. A is equal to B. B is equal to C. Therefore A is equal to C.

2. Ravi is taller than Kannan. Suresh is taller than Ravi. Therefore, Suresh is taller than Kannan.

3. A is the father of B. Therefore B is the father of A.

4. Ravana is the brother of Surpanaka. Therefore Surpanaka is the brother of Ravana.

5. Dasaratha is the father of Rama. Rama is the father of Lava. Therefore, Dasaratha is the father of Lava.

6. A is different from B. B is different from C. Therefore, A is different from C.

7. Nehru was a contemporary of Nasser. Nasser was a contemporary of Kennedy. Therefore, Nehru was a contemporary of Kennedy.

8. Ram owes money to Shyam. Shyam owes money to Rahim. Therefore, Ram owes money to Rahim.

9. Rama is the son of Dasaratha. Rama is the husband of Sita. Therefore Dasaratha is the father-in-law of Sita.

10. I like my son. My son likes ice-cream. Therefore, I like ice-cream.

Questions

1. Give two examples of each of the following :
   (a) many-one relation.
   (b) asymmetrical relation.
   (c) non-transitive relation.
   (d) intransitive symmetrical relation.

2. With reference to relations on what properties do immediate and immediate inferences depend ? Explain with examples.

3 Give an example of each of the relations listed below and assign the logical properties of the relation in each case:

> greater than, twin of, ancestor, married to, equal to, factor to, aunt of, lover of, before, as old as, hates, west of, next to.

4. What are relations ? Classify relations on the basis of their formal properties. Give examples.

Chapter XII

VENN DIAGRAM REPRESENTATION OF CATEGORICAL PROPOSITIONS

The use of circles to represent the meaning of propositions or classes is known as Venn diagram. It is called the Venn diagram after the 19th century English mathematician-logician, John Venn (1834-1923).

Propositions A, E, I and O of traditional logic can be represented by means of Venn diagrams.

Section 1. How to construct a Venn diagram?

First draw a rectangle frame. Then within the frame draw two overlapping circles, naming the left circle S and the right circle P. For ready reference number the regions within the circles and outside the circles.

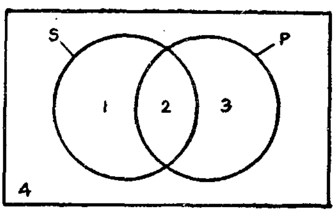


Fig. 20

Region 1 is inside of circle S and outside of circle P.

Region 2 is inside of both circles S and P.

Region 3 is inside of circle P and outside of circle S.

Region 4 is outside of both circles S and P.

Now let us represent A, E, I and O proposition by means of Venn diagrams.

Proposition A

All | monkeys (S) | are | animals. (P)

This means that there are no Ss that are not Ps.

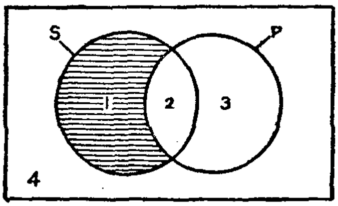


Fig. 21

This diagram shows that no S can exist outside the class of P. This is what is meant by 'All S is P'. That is region 1 is empty or null. We have shown this by shading out region 1 The lines in the Venn diagram mean "there is nothing in the lined area."

Proposition E

No | birds (S) | are | mammals. (P)

This means that there are no objects in the class or section that is both S and P.

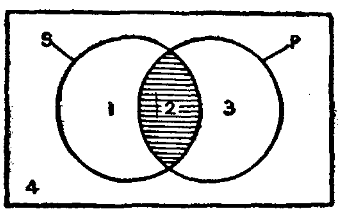


Fig. 22

This diagram shows that nothing can exist in the intersection of S and P. This is what is meant by 'No S is P.' That is region 2 is empty or null.

In short, the lined or shaded portions in circles representing A and E propositions are devoid of members and hence are called *null classes.*

Proposition I

Some | flowers (S) | are | red objects. (P)

This means that there is at least one S which is a P.

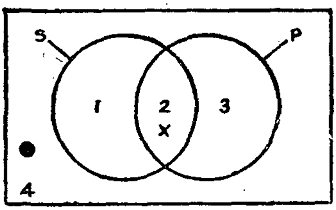


Fig. 23

This means that some members of class S are also members of class P. That is region 2 is not empty. This we show by placing an X in region 2. The X in the intersection shows that there is at least one object (or one X) in this class of S and P.

Proposition O

Some | men (S) | are not | teachers. (P)

This means that at least one S is not a P

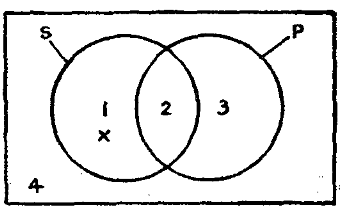


Fig. 24

This means that at least one member of class S is not a member of class P. That is Region 1 falls outside circle P.

Section 2. Venn diagram for testing the validity of conversion.

Proposition A

All | monkeys (S) | are | animals. (P)

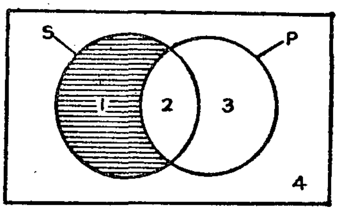


Fig. 25 A

Conversion of proposition A

All | animals (P) | are | monkeys. (S)

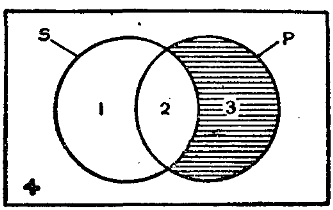


Fig. 25 B

Diagrams 25A and 25B are incompatible. **Therefore simple** conversion of proposition A is not valid.

Proposition E

No | birds (S) | are | mammals. (P)

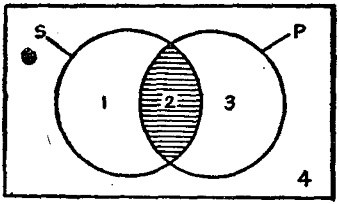


Fig. 26A

Conversion of Proposition E:

No | mammals (P) | are | birds. (S)

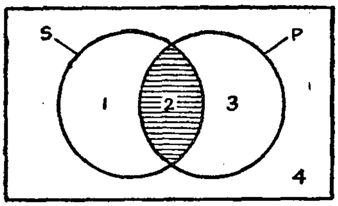


Fig. 26B

Both diagrams 26A and 26B are the same. Therefore conversion of E proposition is valid.

Proposition I

Some | flowers (S) | are | red things. (P)

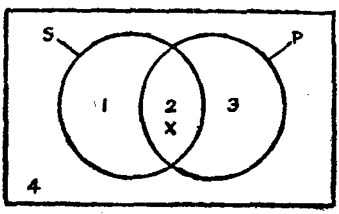


Fig. 27A

Conversion of Proposition I

Some | red things (P) | are | flowers (S).

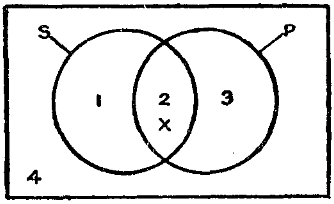


Fig. 27B

Diagrams 27A and 27B are identical and therefore conversion of proposition I is valid.

Proposition O

Some | men (S) | are not | teachers (P).

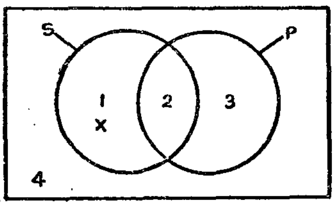


Fig. 28A

If we convert this it would be

Some | teachers (P) | are not | men (S).

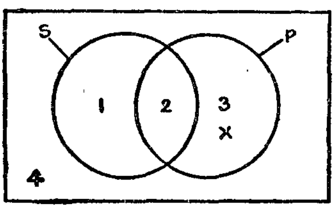


Fig. 28B

Since these two diagrams 28 A and 28 B are different, proposition O has no valid conversion.

Exercises

I. Draw Venn diagrams for the following propositions :

1. No women are mystics.
2. Some writers are conformists.
3. All persons who have short tempers are difficult persons.
4. Some cricketers are not professionals.

II. Show by means of Venn diagrams whether the following immediate inferences are valid or not:

1. All mathematicians are scholars. ∴ All scholars are mathematicians.
2. Men are not infallible. Therefore infallible creatures are not men.
3. Few Indians are Tamilians. Therefore Tamilians are Indians.
4. Few Janata members are khaddar wearers. Therefore a few khaddar wearers are Janata members.

Questions

1. State and explain Venn diagram representation of categorical propositions.
2. Explain how the Venn diagrams are helpful in testing the validity of conversion of propositions.